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January 23, 1995

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AN EMPIRICAL LOOK AT CASH BOND MARKET VOLATILITY IN 1994

OVERVIEW: This report examines the actual price volatilities for all the cash bonds of the U.S. Treasury that traded for the entire calendar year of 1994. The goal was to identify those factors that might explain variations across the yield curve in actual price volatility, with particular emphasis on unusual items, such as the level of Fed holdings of each bond, the bond's strippability, callability, return sensitivity to yield, convexity, *et. al.* A multiple regression model was developed, which appears to explain variations across the yield curve in actual price volatility with exceptional accuracy. A worked example, using a currently trading Treasury bond, is offered to show the relative contributions to actual price volatility of the explanatory factors in the model. Suggestions for use of the Volatility Model in either a trading or portfolio management environment are also made.

DATA/METHODOLOGY: Price data for the U.S. Treasury yield curve were taken from the daily quote sheets published by the U.S. Federal Reserve. Only marketable cash notes and bonds were considered: bills, nonmarketable debt, foreign denominated issues, futures contracts, and zero-coupon strips were not evaluated. Also, any Treasury issue that matured during 1994, or was first issued during 1994, was excluded from study. A total of 178 securities were thus available for study, each of which had 248 trading-day price observations. Unfortunately, dropping all newly issued bonds from the sample precluded analysis of any "on-the-run" effect, but was necessary due to the overwhelming distortions imposed by their inclusion. Nevertheless, valuable results were obtained by a study limited to the "off-the-run" issues.

Each price observation was calculated as the mid-point of the bid and ask clean prices from the Fed quote sheets, with accrued interest added back into this clean mid-price. Volatility for each instrument was calculated as the sample standard deviation of the daily log price ratios (the traditional method used in the financial markets) across the entire 248-day sample period. No special adjustments to the standard deviation calculation were made for weekends or holidays. Similarly, no special attempt was made to model the possibly time-varying nature of the volatility across the year. Variables available for the model were classified into three categories: Defining characteristics of each bond, market characteristics of each bond, and general availability of each bond. The defining characteristics of a bond included its original maturity, remaining maturity, callability, strippability, and coupon rate. The market characteristics included return sensitivity to yield and convexity, both as of the last trading day of the sample. The general availability of a bond included the total amount issued by the Treasury over its lifetime, as well as the level of holdings of that bond by the Federal Reserve.

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ANALYSIS: Graphs of the sample volatilities against several of the different explanatory variables are included at the end of this report. Specific comments about each graph are made below, along with analyses of each of the explanatory variables. Table 2 presents the detailed regression results and diagnostic statistics for the Volatility Model. Overall, the results were quite outstanding. The adjusted R^2 was 99.2%, while the standard error of the fitted model was only 0.23 (interpretation: 95% of the volatilities in our sample were fitted by the Volatility Model within a range of plus or minus 46 basis points, a very impressive fit considering that the average volatility in the sample was 7.25% with a standard deviation of 2.61%). All the variables included had strong t-statistics.

Factors Explaining Volatility:

<u>Original Maturity</u> - As seen in Figure 1, volatility increases roughly with original maturity, although there are breaks in the relationship, particularly for very short and very long maturities. A total of 9 dummy variables was used to model the impact of the original maturity of each bond (2, 3, 5, 7, 10, 15, 20, and 30 year dummies, plus a single dummy to composite 5 instruments that had rare initial maturities). The constant was suppressed from the regression so that all 9 dummy variables could be included (typically one dummy variable must be dropped if a constant is included), and so that each dummy variable's coefficient would represent the conditional average volatility for that original maturity. According to the coefficient estimates (see Table 2), **the original maturity of a bond plays a major role in determining the volatility of the bond, accounting for about 15% to 35% of the bond's volatility**. For example, original 15yr maturity bonds have exceptionally high volatilities, while original 10yr and 30yr bonds have relatively low volatilities. The so-called "ODD5" instruments (3 bonds with 25yr original maturities and 2 with 40yr original maturities) trade very quietly, with volatilities lower than any other instrument except 2yr notes.

<u>Remaining Maturity</u> - Figure 2 illustrates a much tighter positive relationship between a bond's remaining time to maturity and its volatility than was seen for original maturity. While the relationship is less strong for very short maturities, it seems very accurate for the longer end of the curve, with deviations from a straight line in the long end for those bonds that are either callable or strippable. Since remaining maturity and return sensitivity are so closely related, however, it is difficult to say for sure based just on visual inspection which explanatory variable (remaining maturity or return sensitivity) gives the graph in Figure 2 its positive slope. To allow for possible curvature of the relationship between time remaining and volatility, both the absolute time remaining as well as the natural logarithm of time remaining were included as explanatory variables. While remaining time to maturity had a positive coefficient, the natural logarithm had a negative influence. For certain intermediate maturities, this leads to the result that time remaining to maturity can have a negative influence on volatility. **For most maturity ranges, however, time remaining to maturity has a positive influence on volatility, an influence which varies according to the remaining volatility itself** (due to the non-linear impact of the logarithmic function).

<u>Coupon Rate</u> - As can be seen in Figure 3, there is a very tentative positive relationship between a bond's coupon rate and its price volatility, albeit with what appears to be numerous exceptions for above-average coupon levels. Interestingly, the relationship between coupon rate and volatility appears to break down for bonds with coupons of 8% or more, the pivotal coupon level in the determination of deliverability conversion factors into the bond futures contract. While the relationship between coupon rate and price volatility appears tenuous on a 2-dimensional graph, the relationship becomes much more accurate once other factors are controlled for via multiple regression. According to the Volatility Model, **for every 100 bps of additional coupon income a bond pays, it suffers 31 bps of additional volatility**. The explanation is similar to that given below for strippability: high coupon bonds have a special market segment that finds them appealing solely for the purpose of current income. As such, the enhanced market attention leads to added supply and demand pressures, which adds to volatility.

Return Sensitivity to Yield - As mentioned above in the discussion of remaining maturity as a factor, simple 2-dimensional graphs show that both remaining maturity and return sensitivity to yield bear a positive visual relationship to volatility. (Return sensitivity to yield is defined here as the ratio of the dollar price sensitivity per basis point over original invoice price). A graph of the relationship between return sensitivity and volatility can be seen in Figure 4. Similar to Figure 1, the relationship in Figure 4 appears more accurate with higher values of return sensitivity. According to the regression results, **for every .01 increase in the return sensitivity measure, volatility increases by 71 bps**. Clearly there is a strong relationship between the mathematical sensitivity of a bond to a potential change in yield and the actual statistical sensitivity observed in the market.

Strippability - *Ceteris paribus* in the Volatility Model, **if a bond is strippable it experiences an extra 26 bps of volatility**. While there might be several more sophisticated arguments for this, the simplest explanation is that strippable notes and bonds receive a tremendous amount of attention in the markets, being bought and sold for all the reasons of similar maturity instruments plus for the ability to strip their coupons. As such, the added supply and demand pressures add to the volatility of these instruments.

Callability - *Ceteris paribus* in the Volatility Model, **the callability of a Treasury trims about 104 bps from a bond's volatility**. Reversing the logic of strippability, callability is a feature entirely benefiting the original issuer, and of no significant benefit to either the buyer or seller in the secondary market. As such, it appears the market avoids active trading of callable issues, *ceteris paribus*, reducing volatility.

Fed Holdings - Based on the assumption that the more the Fed buys of an outstanding issue the less there is for the market to trade (and perhaps thus creating a heightened trading environment for what remains, with higher volatility), an explanatory variable was added for the ratio of the Fed's holdings of an issue to the total amount issued by the Treasury. According to our model, **for every 1 percentage point of the outstanding issue the Fed acquires, the volatility of that issue rises 1.3 bps**.

Convexity - The model shows that, *ceteris paribus*, bonds with higher dollar convexity experience lower volatility. Quantifying that, we see that: **for every 10 cent increase in convexity (per 1 bp change per $1 Million par value), the volatility of that issue falls 90 bps**. It may appear to be a contradiction that higher convexity is associated with lower volatility in the model, since bond analysts generally recommend buying high convexity bonds to take advantage of anticipated jumps in volatility. In fact, the delivery of lower volatility by the market to bonds which have higher convexity is the natural outcome of an efficient bond market, which is trying to offset the advantage of higher convexity under high volatility scenarios.

Failed Factors - Surprisingly, a number of factors that might have been viewed as *de facto* influences on volatility were not. For example, the total outstanding amount of each issue had little explanatory power on the issue's volatility, and neither did the outstanding amount of Fed holdings (although the ratio of Fed holdings to total outstanding did serve as a useful variable). While the original maturity served as a valuable predictor of volatility (as did the time remaining to maturity), the percentage of time remaining on the original maturity was not a useful variable.

Most fascinating, however, was the distorting impact of including the volatilities of those instruments that were newly issued during 1994. Every effort was made to model the impact of these securities, since they constitute the so-called "on-the-run" securities of 1994, and are of greatest importance to investors and traders. Unfortunately, the on-the-run observations distorted most of the regression estimators when mixed with the sample of off-the-run securities. For those who would be interested, however, the following should be noted: when the on-the-run issues were included in the sample and a dummy variable was incorporated to model their unique status, it showed that on-the-run securities generally had lower volatility, *ceteris* *paribus*, than other issues on the curve. This runs counter to the arguments made above for strippability and high coupon levels (that added attention brings added volatility), since on-the-run securities are the most closely watched and actively traded instruments on the curve. In all likelihood, the small sample sizes of the price data for some of these on-the-run instruments (in particular, those issued later in 1994), as well as the fact that true volatility may have varied over the year in an unpredictable way, precluded any consistent use of the on-the-run securities in the Volatility Model.

WORKED EXAMPLE: To show the impact of each explanatory variable on volatility, a worked example on a typical bond in the data set is provided below. The bond is an original 30 year maturity, strippable but not callable, with an 8% coupon and a maturity date of 11/15/2021. As can be seen, the true volatility of this instrument is 12.324%, while the predicted value was 12.449%, an error rate of less than 7 bps.

TABLE 1:
APPLICATION OF THE VOLATILITY MODEL

VARIABLE	LEVEL	COEFF	ADDS TO VOL
Defining Characteristics:			
Original Maturity	30 Years	1.4563	1.4563
Remaining Maturity	26.88 Years	0.1316	3.5369
LN(Remaining Maturity)	3.2913	-0.4126	-1.3580
Callable	No	-1.0361	0.000
Strippable	Yes	0.2603	0.2603
Coupon	8.000%	0.3126	2.5007
Market Characteristics:			
Return Sensitivity	0.1100	70.6524	7.7685
Convexity	$2.00	-0.9014	-1.8029
Supply:			
Fed Holdings	$660		
Total Outstanding	$32,798		
Percent	2.01%	0.0127	0.0256
	PREDICTED:		12.387
	ACTUAL:		12.324
	ERROR:		0.063

As can be seen from the table, the major contributing factor to the bond's volatility is the return sensitivity, trimmed back a bit by its convexity. Together the market characteristics contribute about 5.97% of the total 12.387% volatility, or roughly half. Ranking behind the impact of the market characteristics is the impact of the coupon rate, which in this case adds another 2.50% to volatility. Time remaining to maturity, after adjusting for the non-linear component, appears to add about 2.18%. Original maturity adds another 1.46%. Strippability and the percent held by the Fed add only about 0.29% more, while callability played no role in determining the level of volatility for this bond.

RECOMMENDATIONS: The complete set of regression coefficients for the Volatility Model is presented in Table 2. An Excel spreadsheet of the original data set used to develop the Volatility Model is available from the author upon request. The following are some potential uses of the Volatility Model:

-1- To appreciate the reasons for differing volatility amongst seemingly similar instruments. For example, in our data set there are two notes due Aug 15 2000, both with very similar coupons (8 3/8% vs. 8 3/4%), but with volatilities that differ by almost 50 bps. Knowing that differences such as original maturity (10yrs vs. 25yrs), differences in the percent held by the Fed (5% vs. 48%), and differences in callability and strippability can account for such a difference might prevent an analyst from concluding inappropriately that he had discovered a risk-free, exploitable anomaly on the curve.

-2- To provide "what-if" analyses of the impact on volatility of changes in the explanatory variables. For example, the Model offers a method for predicting how much volatility should change with the passage of time, or with a change in return sensitivity , convexity, or Fed holdings.

-3- To provide for the evaluation of whether actual volatility differs from theoretical volatility. Traders who trade options based on volatility can perform a "rich/cheap" analysis of volatility based on the residuals from the Volatility Model.

-4- To estimate the volatility of newly created fixed income instruments. Whether the creation of the instrument results from the new issue of a bond by the Treasury or by the combination of strips into a new synthetic, the Volatility Model provides a basis for predicting the type of volatility to expect once the instrument starts trading.

-5- To provide a framework for re-estimation. Assuming that the overall level of volatility in the market can shift over time, the specific parameters offered in the Model here may not remain accurate over time. Nonetheless, the structure of the Model may remain valid, requiring only simple re-estimation of the structural form in order to implement the strategies listed above.

TABLE 2: REGRESSION ESTIMATES CASH BOND MARKET VOLATILITY MODEL		
VARIABLE	COEFFICIENT	T-STATISTIC
ORIGINAL MATURITY:		
2 YEAR	0.943440	6.333877
3 YEAR	1.159092	6.442571
5 YEAR	1.977286	9.523453
7 YEAR	2.185075	8.937767
10 YEAR	2.022177	6.575499
15 YEAR	2.927952	9.316256
20 YEAR	1.523478	4.093747
30 YEAR	1.456284	3.960455
ODD5	0.987309	3.302832
CALLABLE	-1.036127	-7.340854
STRIPPABLE	0.260342	2.182473
COUPON RATE	0.312583	13.90313
RETURN SENSITIVITY	70.65240	11.97322
FED HOLDINGS	0.012706	3.206593
CONVEXITY	-0.901356	-2.365398
YEARS TO MATURITY	0.131592	3.961834
LN(YEARS TO MATURITY)	-0.412603	-8.559387
R-SQUARED	99.29%	
ADJUSTED R-SQUARED	99.22%	
STANDARD ERROR	0.229584	
DURBIN-WATSON	1.301956	

FIGURE 1: BOND VOLATILITY
VS. ORIGINAL MATURITY

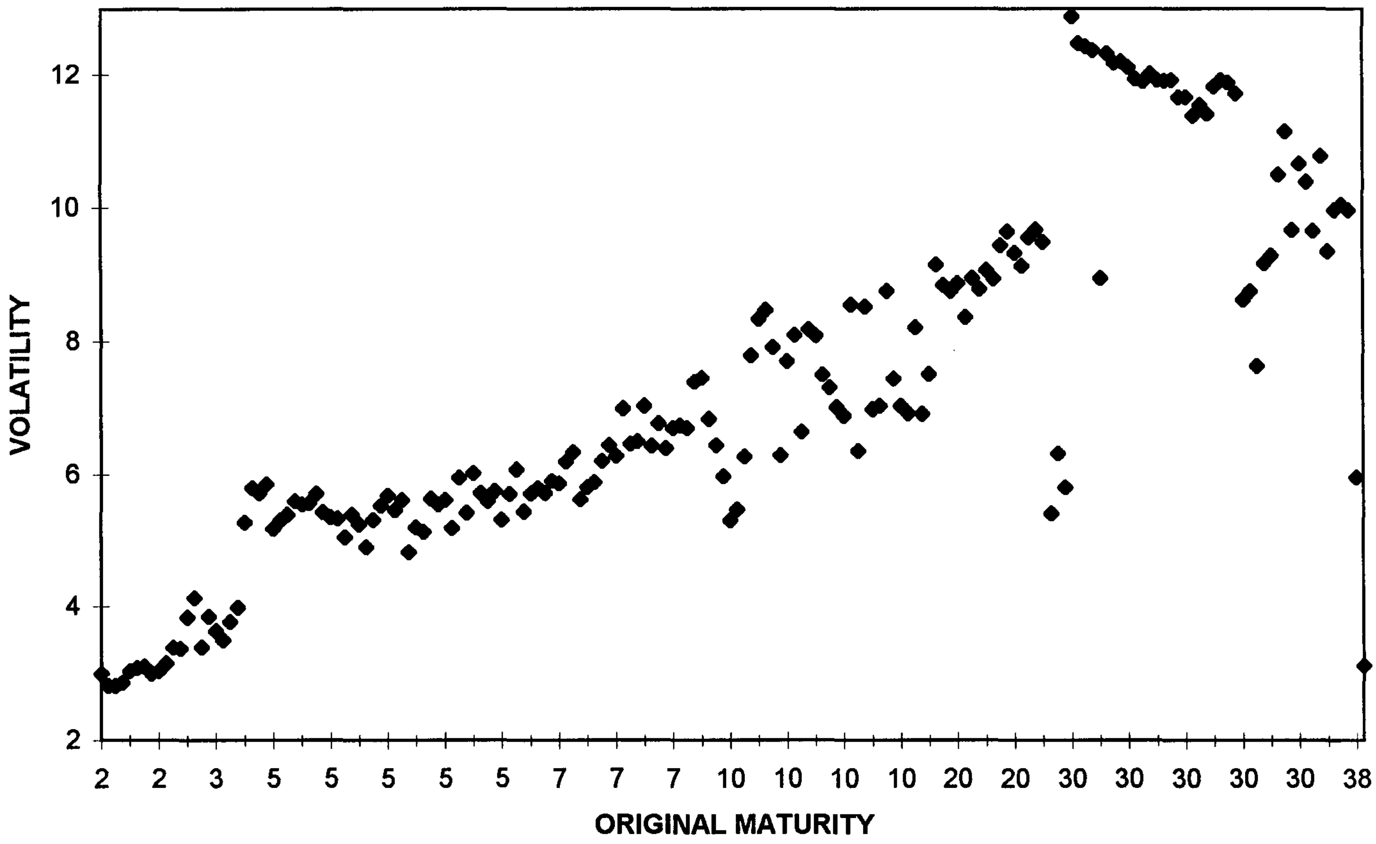

FIGURE 2: BOND VOLATILITY
VS. REMAINING MATURITY

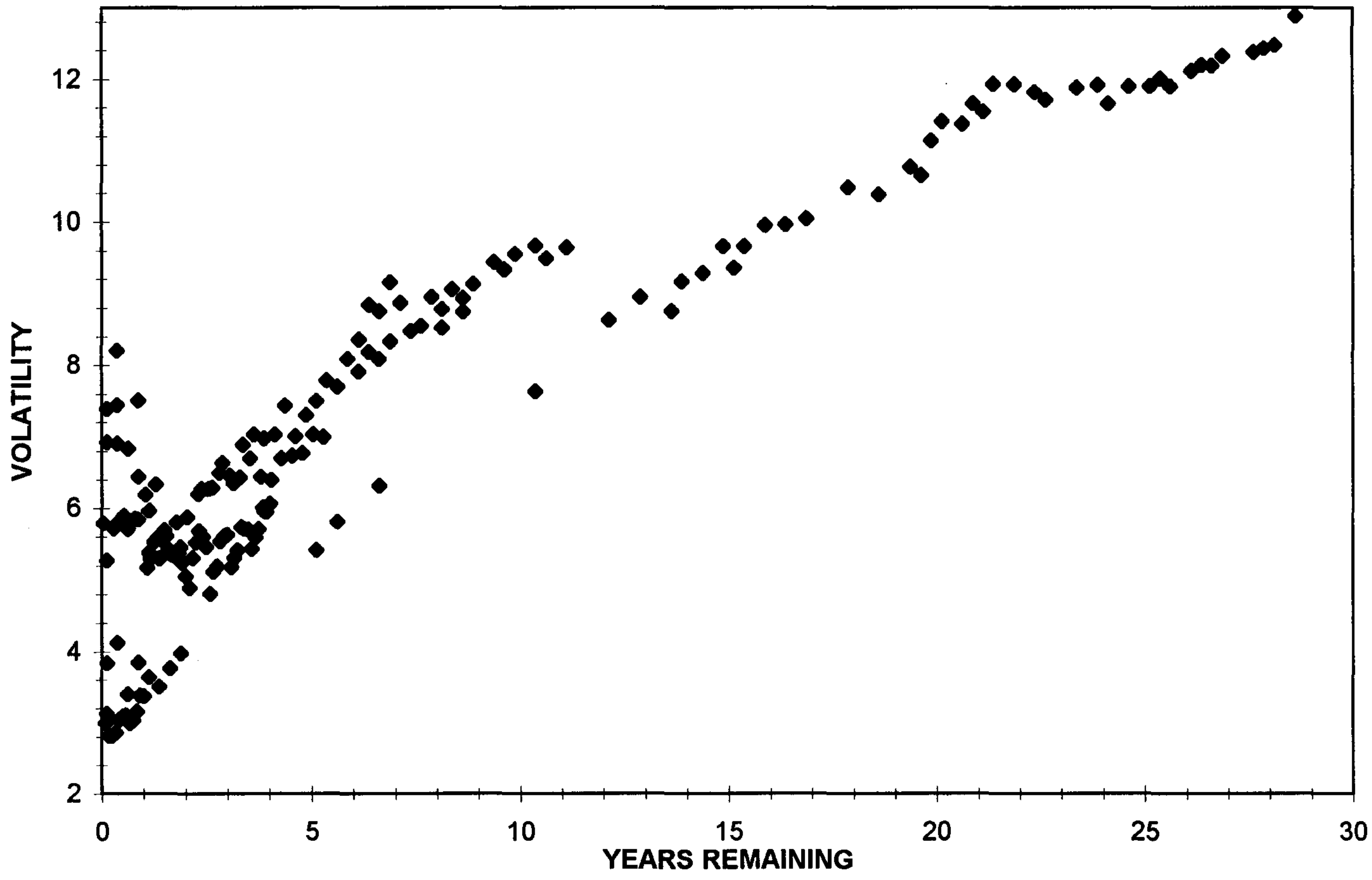

FIGURE 3: BOND VOLATILITY
VS. COUPON RATE

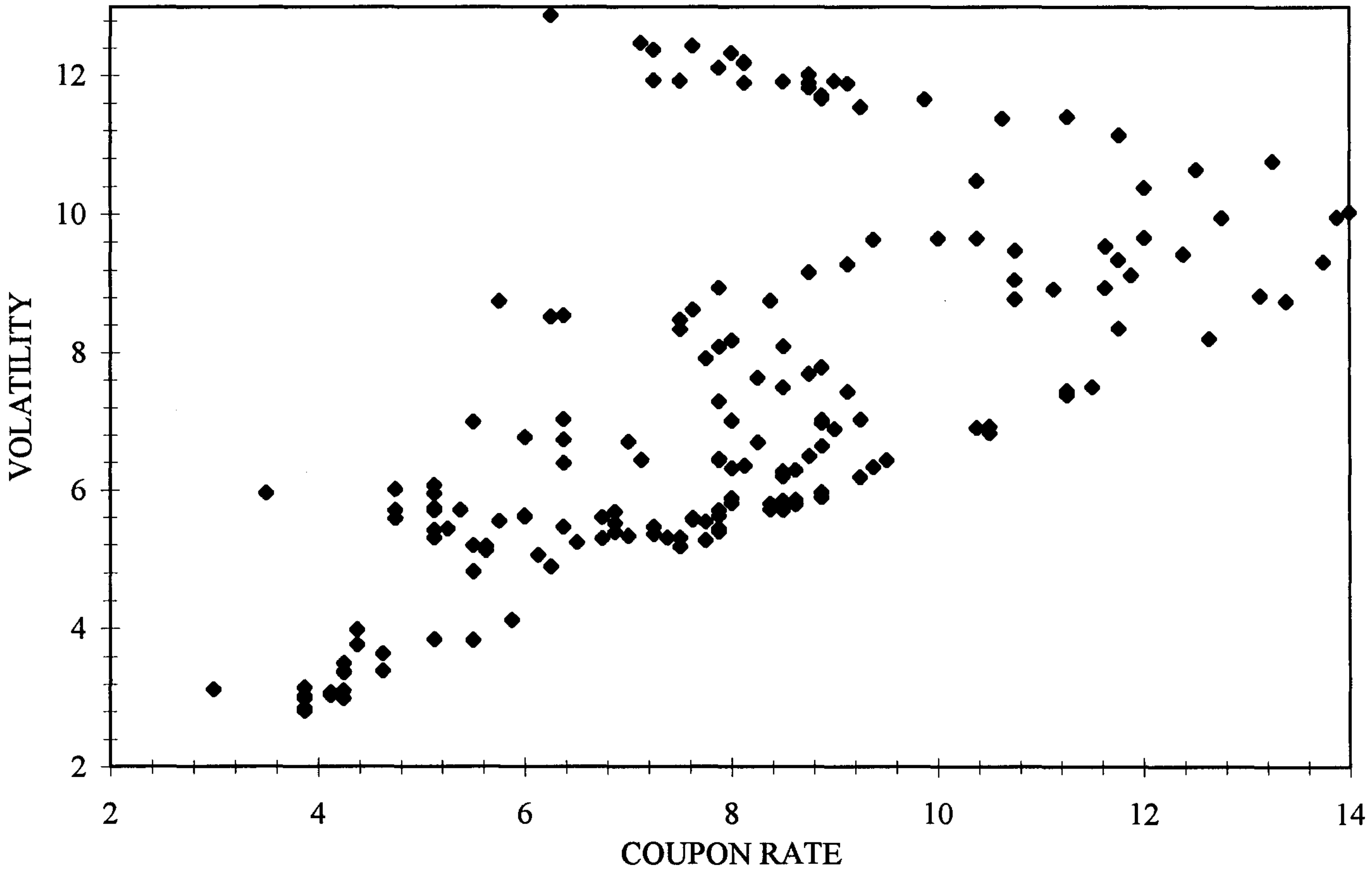

FIGURE 4: BOND VOLATILITY
VS. RETURN SENSITIVITY