CFA Level II Review
Study Session 18: Options

USC/LASFA CFA Review Program

USC Campus

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SANWA BANK CALIFORNIA

SESSION OVERVIEW

0. **Preliminary Material**

1. **Options**
   - A. **Option Payoffs and Option Strategies**
   - B. **European Option Pricing**
   - C. **Option Sensitivities and Option Hedging**
   - D. **Options on Indices, Currencies, and Futures**

2. **Interest Rate Derivative Instruments**

3. **Option Models and Option Prices**

PRELIMINARY MATERIAL

Preliminary Material

Option Definitions (Kolb p. 281)

	Basic Option Mechanics	
	Buyer has the:	**Seller has the:**
Call Option	**RIGHT to Buy the Asset**	**OBLIGATION to Sell the Asset**
Put Option	**RIGHT to Sell the Asset**	**OBLIGATION to Buy the Asset**

NOTE: Futures only have Obligations, Options have Rights and Obligations!!

Preliminary Material

Option Definitions (Kolb pp. 282-283)

Important Features of Options

1. **Exercise Price (X) -** The price at which the option buyer can force the option seller to transact. aka the "strike" price

2. **Option Premium (c or p) -** The price the option buyer pays, and the option seller receives, for the option contract.

3. **Time to Expiration (t) -** The period of time over which the option buyer has the right to compel the seller to transact.

4. **Underlying Asset (S) -** The asset that the option buyer and seller agree is the asset they will trade and/or track in value.

5. **Early Exercise -** Options can be exercised any time before expiration (American style) or only at expiration (European style).

Preliminary Material

Option Definitions (Kolb pp. 287-291)

Possible Underlying Assets for Options

- **Stock Options** - Options on individual stocks, usually defined on a specific number of shares to be delivered if exercised.
- **Index Options** - Options that let you buy or sell an index times a multiplier at the cost of the strike price times the multiplier
- **Foreign Currency Options** - Options that deliver a fixed amount of a foreign currency
- **Options on Futures** - options which, if exercised, put the option buyer and seller in offsetting long/short positions in an exchange-traded futures contract

Preliminary Material

Moneyness (Kolb pp. 282-283)

"Moneyness" of Options

	Deep In The Money	In the Money	At the Money	Out of the Money	Deep Out of the Money
Call Option	S >> X	S > X	S = X	S < X	S << X
Put Option	S << X	S < X	S = X	S > X	S >> X

Preliminary Material

Intrinsic Value & Moneyness (Kolb pp. 308-316)

Intrinsic Value

The gross value of an option (i.e., not considering its purchase price) if it were to be exercised immediately. An option has intrinsic value if it is in-the-money, and has no intrinsic value if it is out-of-the-money.

Intrinsic Value Formulas

CALL: $C_t = MAX\{0, S_t - X\}$

PUT: $P_t = MAX\{0, X - S_t\}$

Preliminary Material

Intrinsic Value (Kolb pp. 308-316)

<u>Intrinsic Value Examples</u>

1. A Call option has a strike price of \$105 and the underlying asset is selling for \$115. What is the call's intrinsic value?

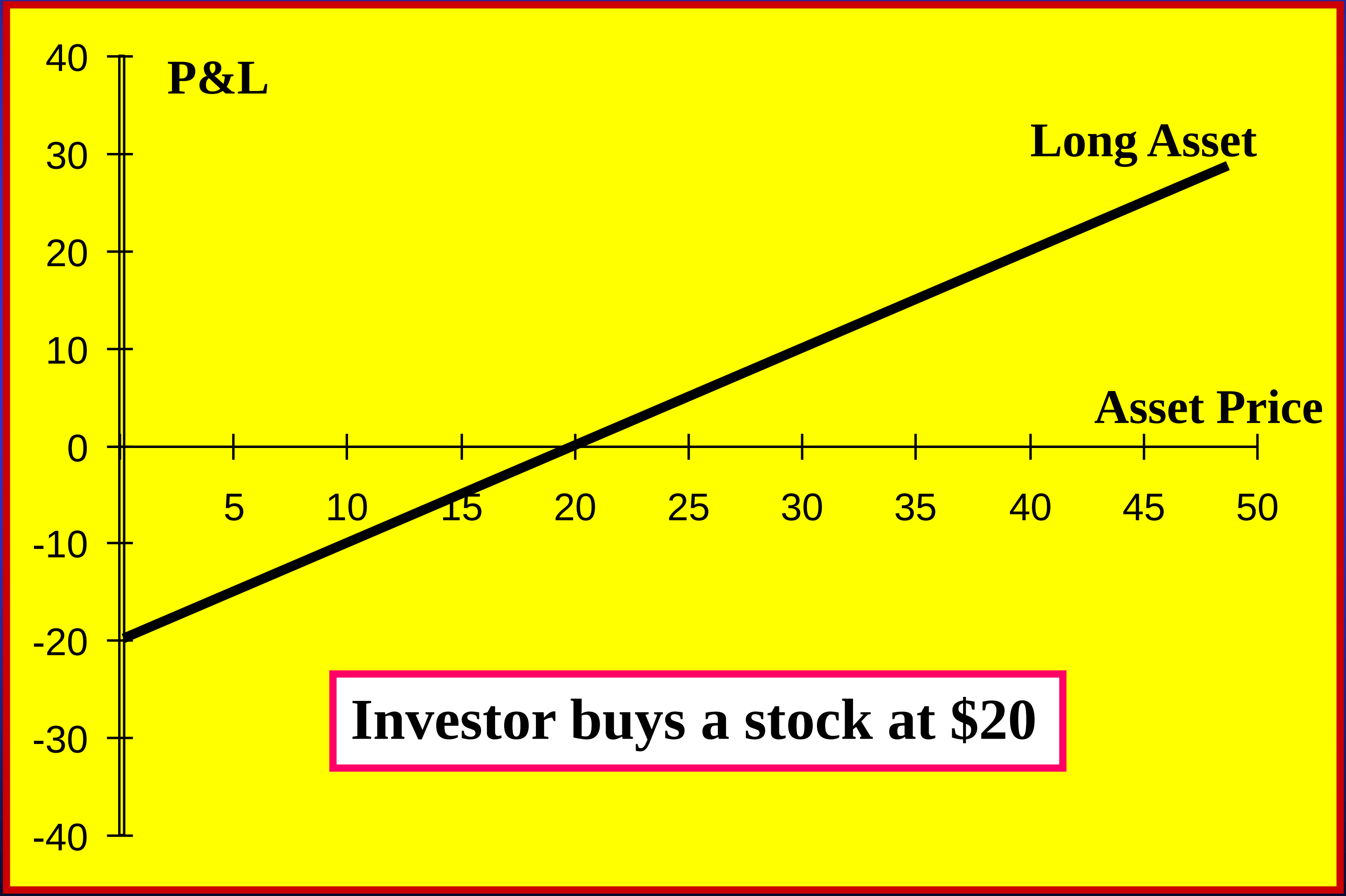
$$\mathbf{C_t = MAX\{0, \$115 - \$105\} = MAX\{0, \$10\} = \$10}$$

2. A Put option is selling for \$7, has three months to maturity, has a strike price of \$15, and the underlying asset is selling for \$25. What is the put's intrinsic value?

$$\mathbf{P_T = MAX\{0, \$15 - \$25\} = MAX\{0, -\$10\} = \$0}$$

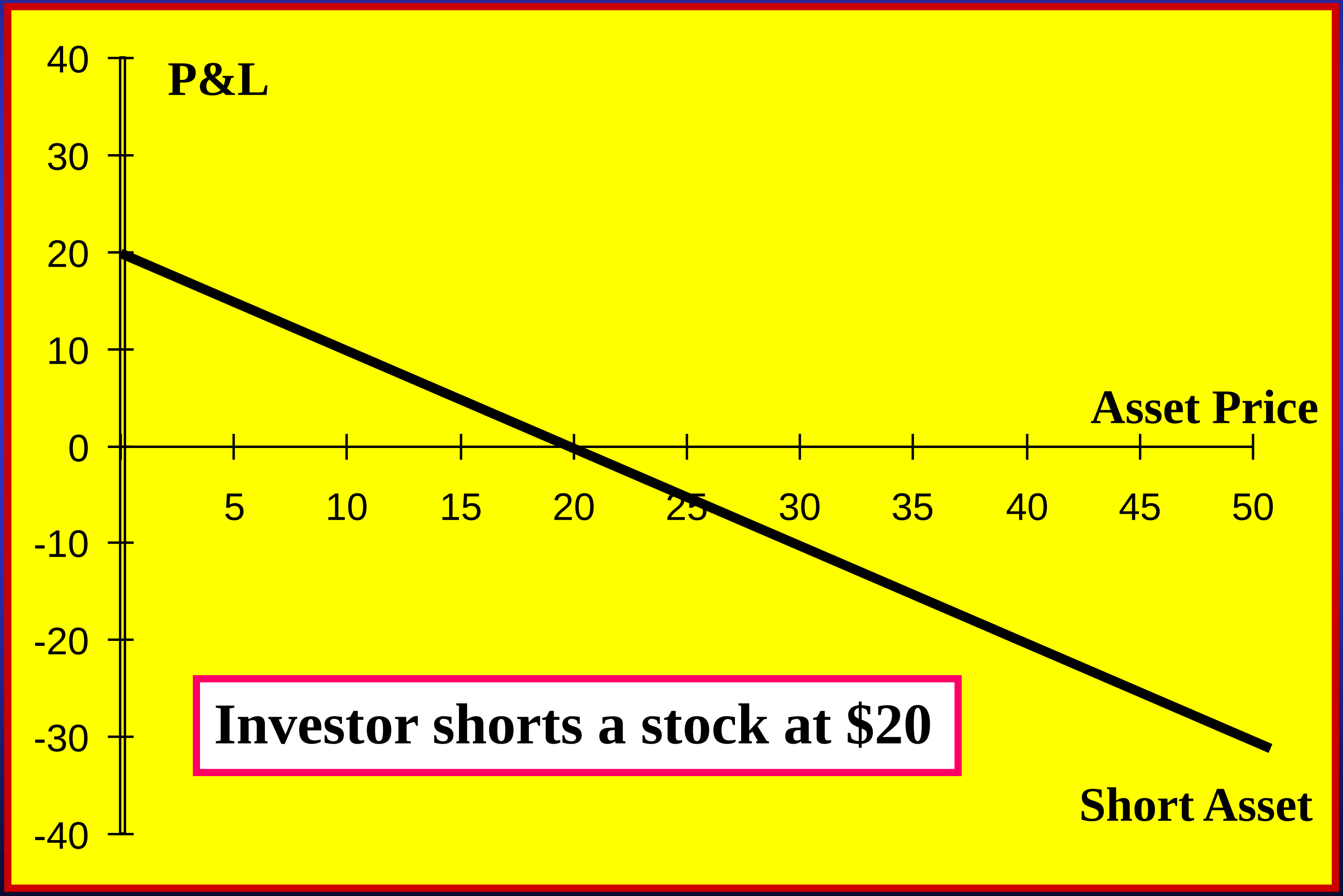
Preliminary Material

Payoff Diagrams - Long Asset (Kolb pp. 305-306)



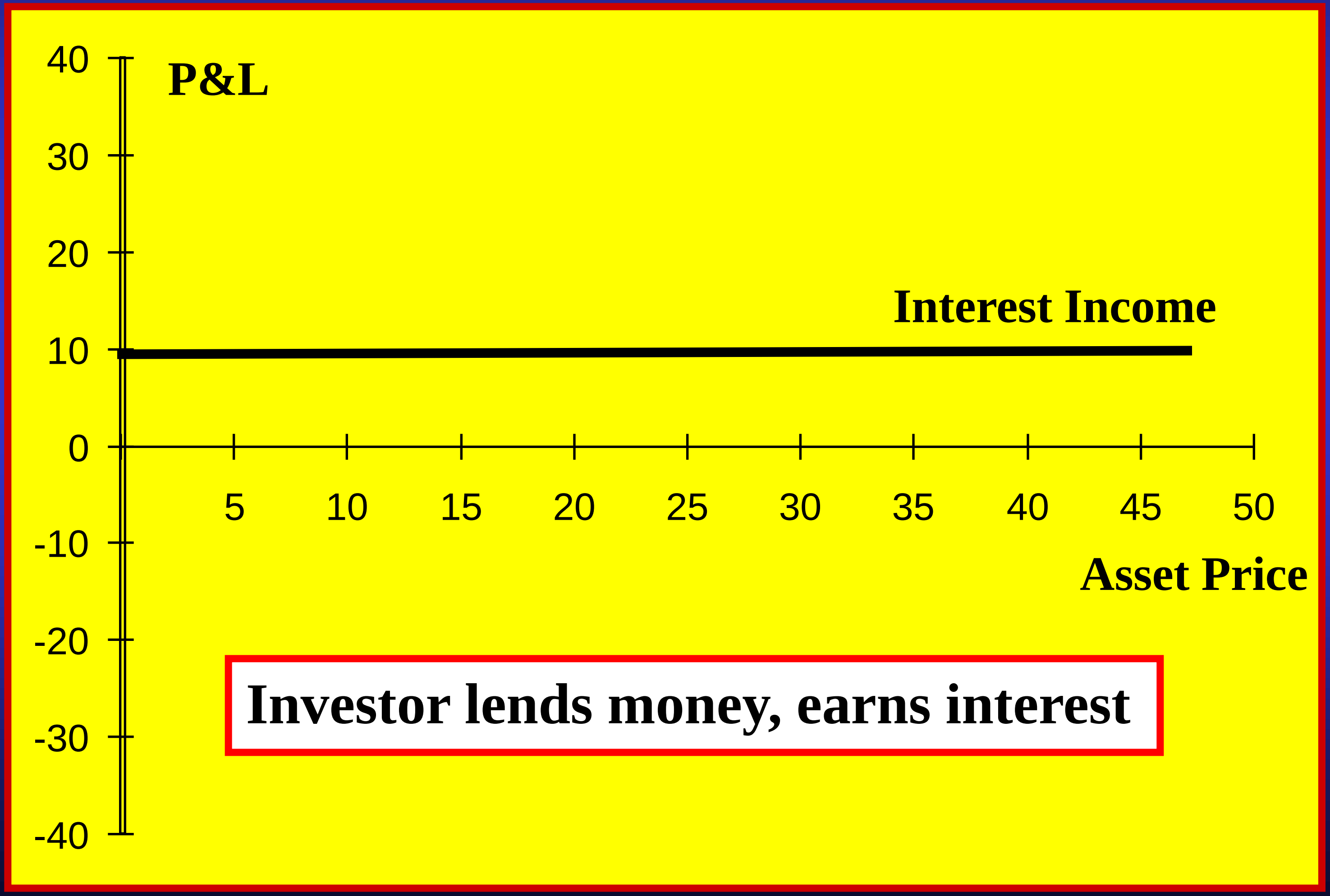
Preliminary Material

Payoff Diagrams - Short Asset (Kolb pp. 305-306)



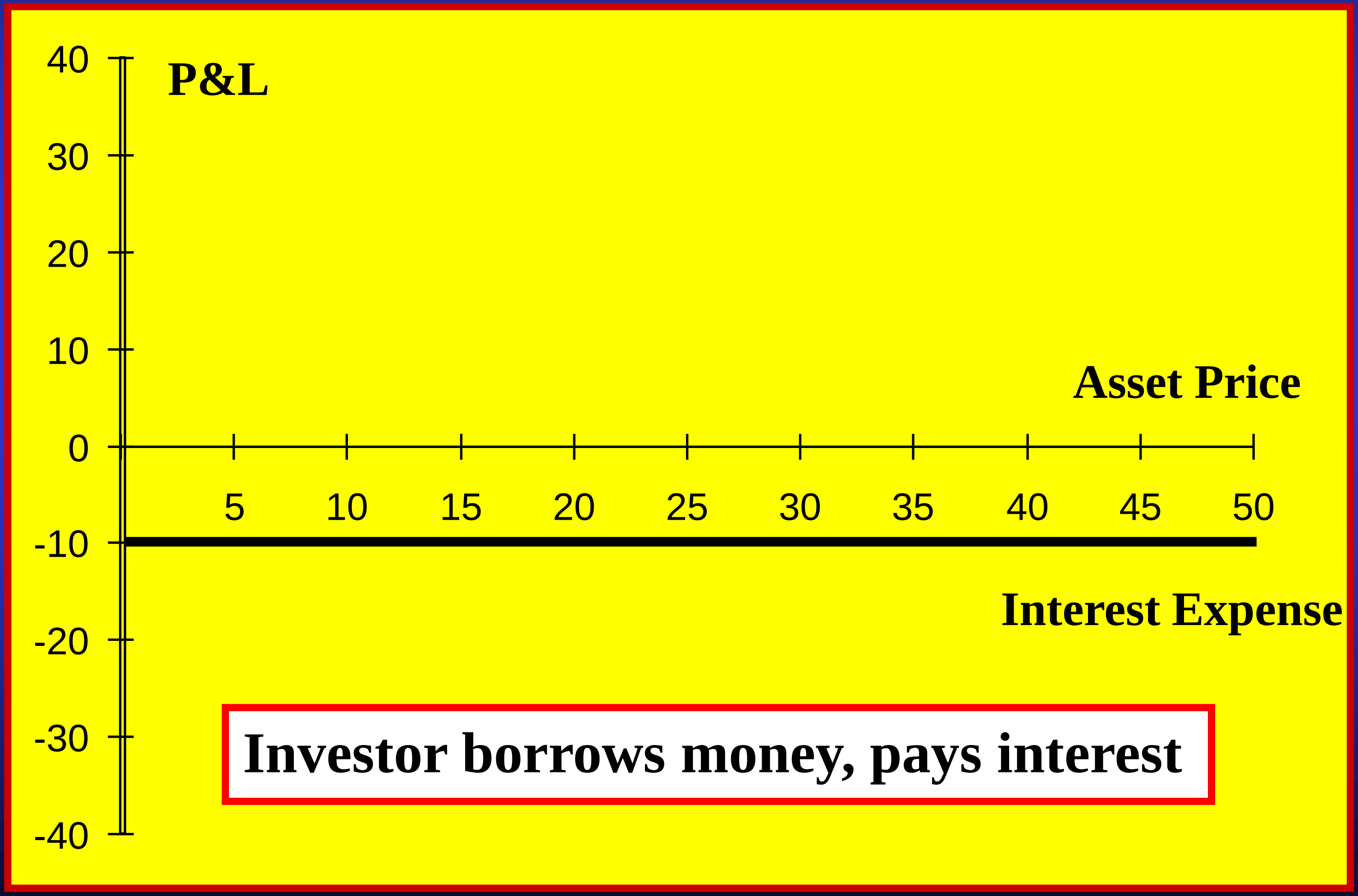
Preliminary Material

Payoff Diagram - Long Bond (Kolb pp. 306-307)



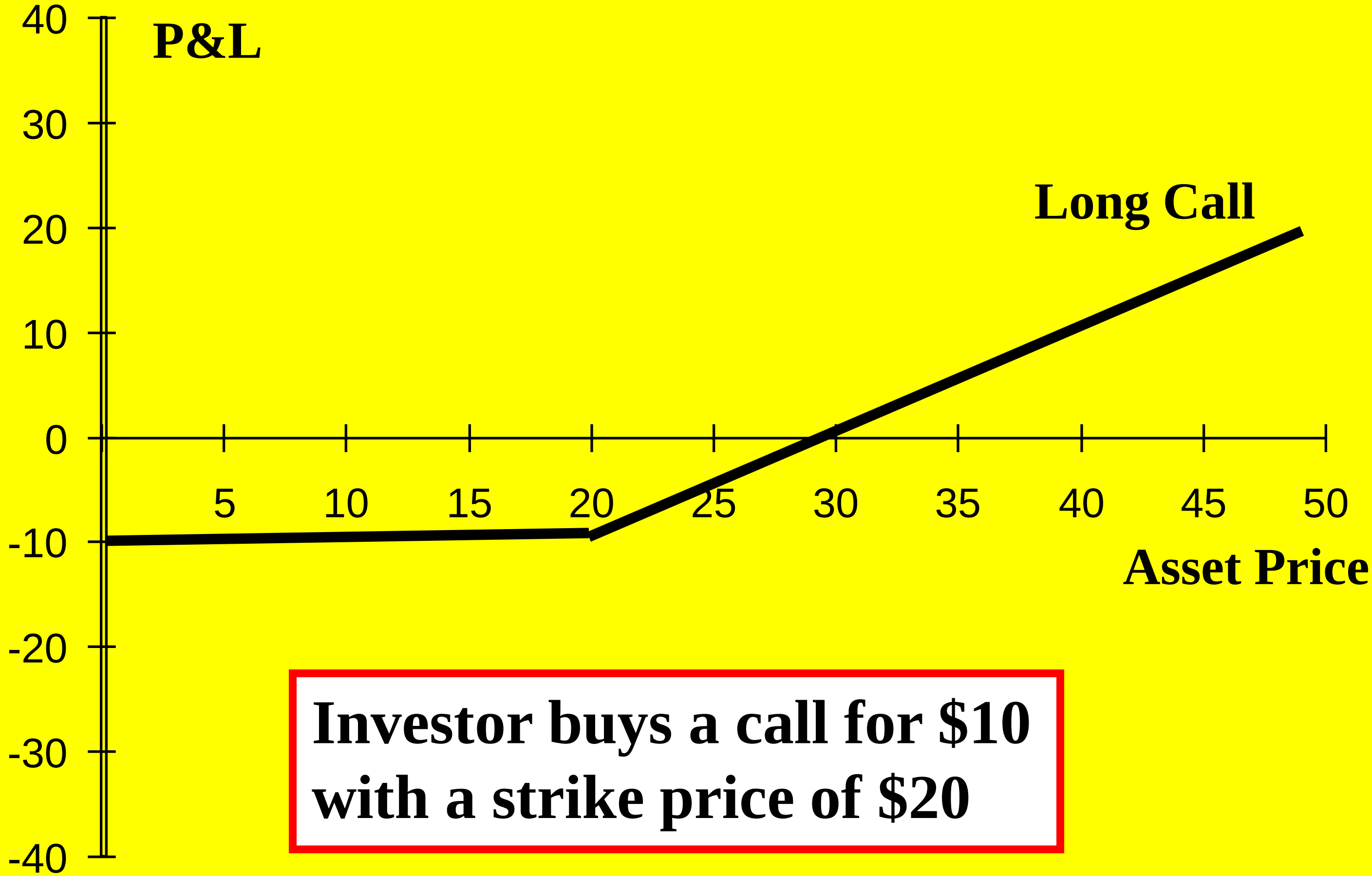
Preliminary Material

Payoff Diagrams - Short Bond (Kolb pp. 306-307)



Preliminary Material

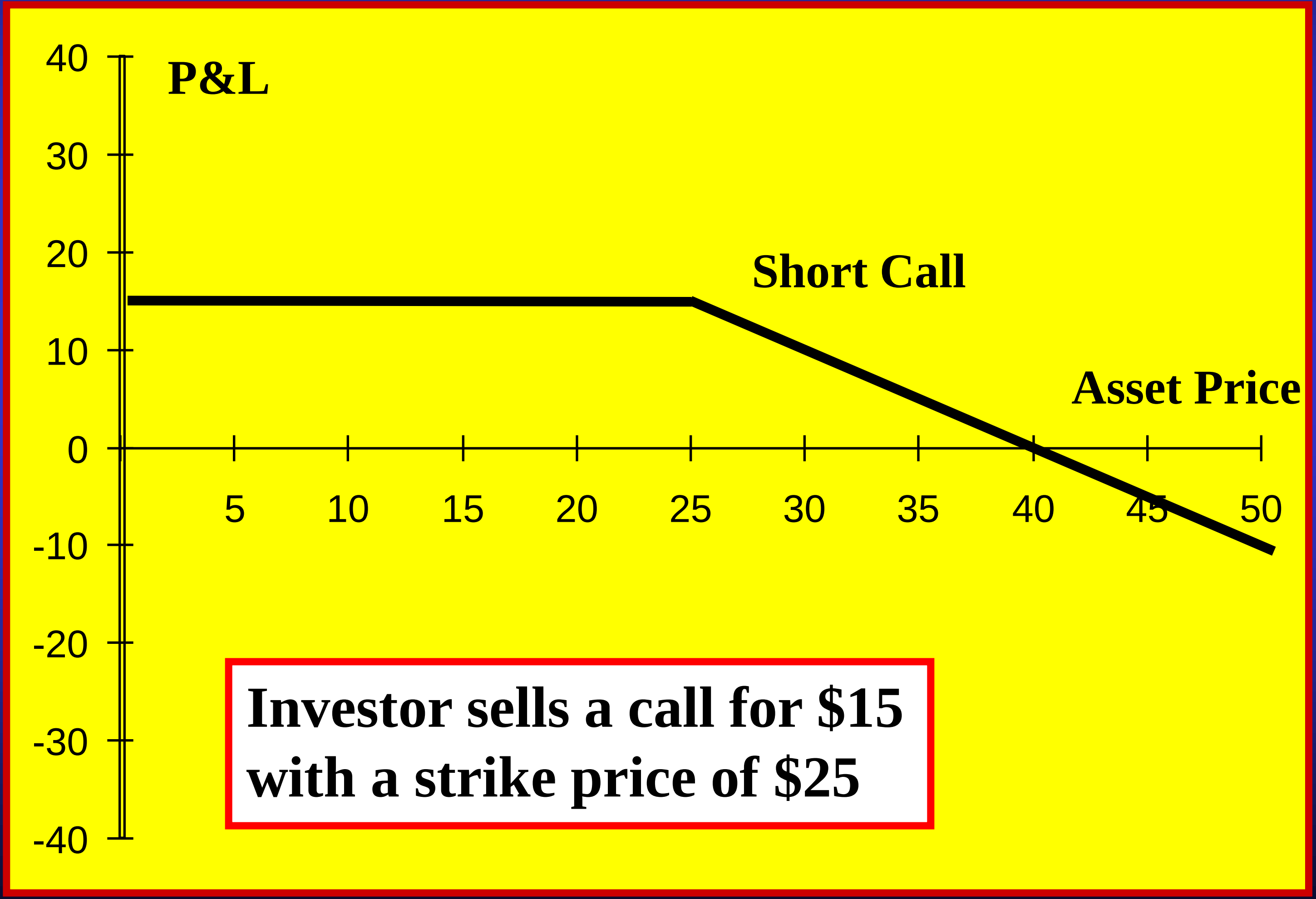
Payoff Diagram - Long Call (Kolb pp. 310-311)



Investor buys a call for $10 with a strike price of $20

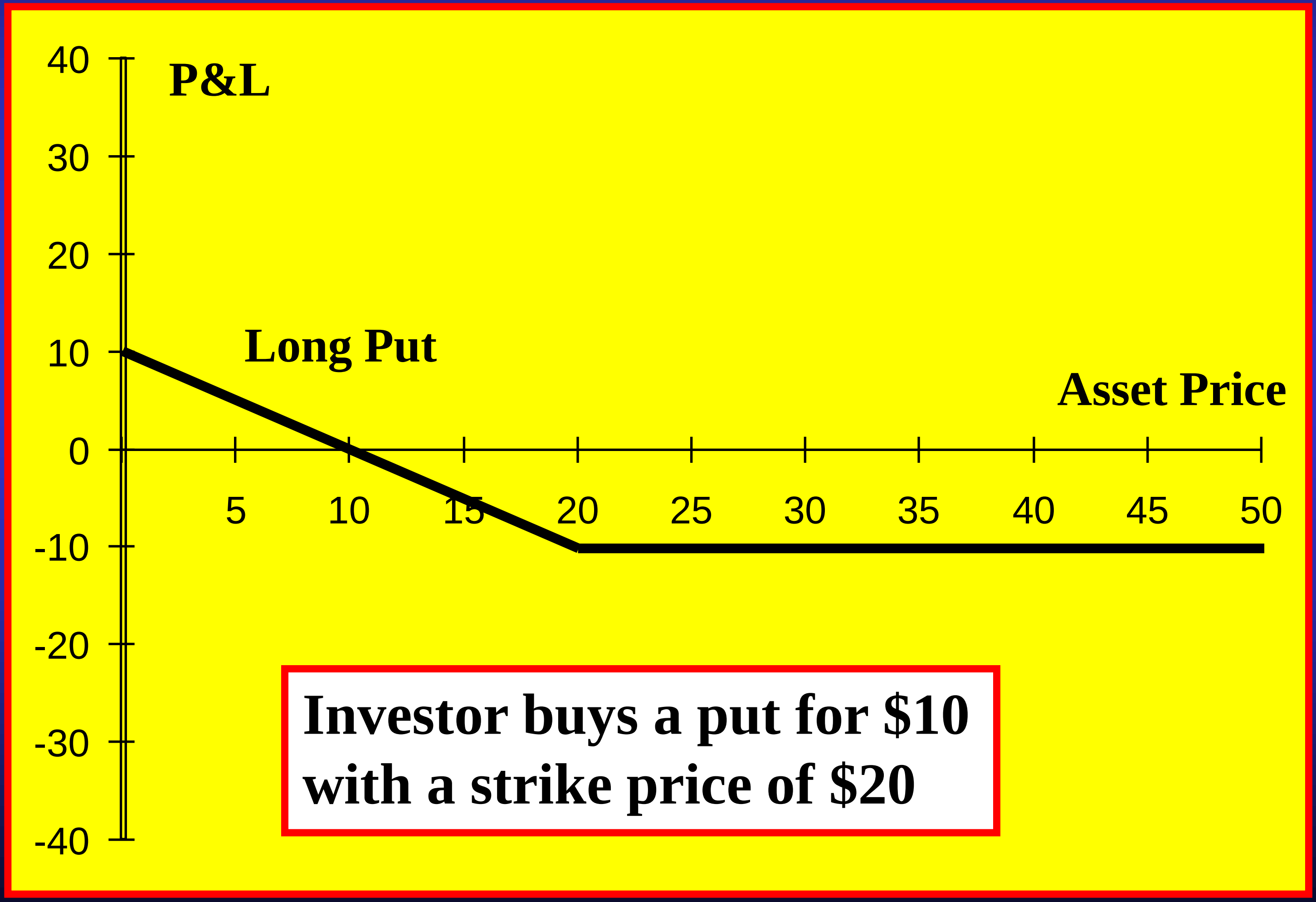
Preliminary Material

Payoff Diagram - Short Call (Kolb pp. 310-311)



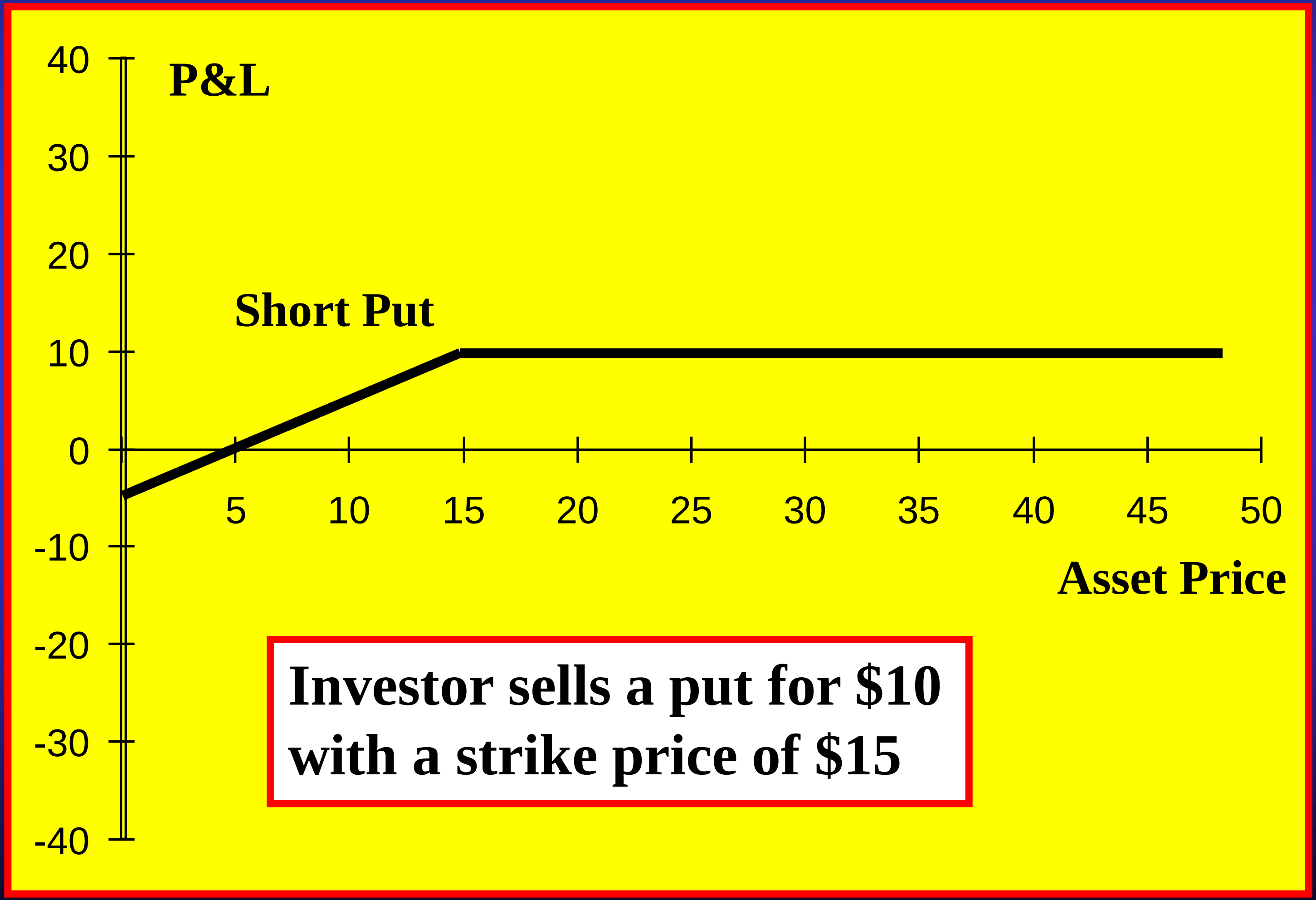
Preliminary Material

Payoff Diagram - Long Put (Kolb pp. 313-315)



Preliminary Material

Payoff Diagram - Short Put (Kolb pp. 313-315)



OPTION PAYOFFS AND OPTION STRATEGIES

Option Payoffs/Strategies

LOS 1 A a), b) c) Long Straddles (Kolb pp. 316-318)

Definition

- A **Long Straddle** consists of a long position in a put and a call, both with the same strike price and the same expiration.
- Used generally as a bet on volatility rather than on direction

Cost Paid/Premium Received

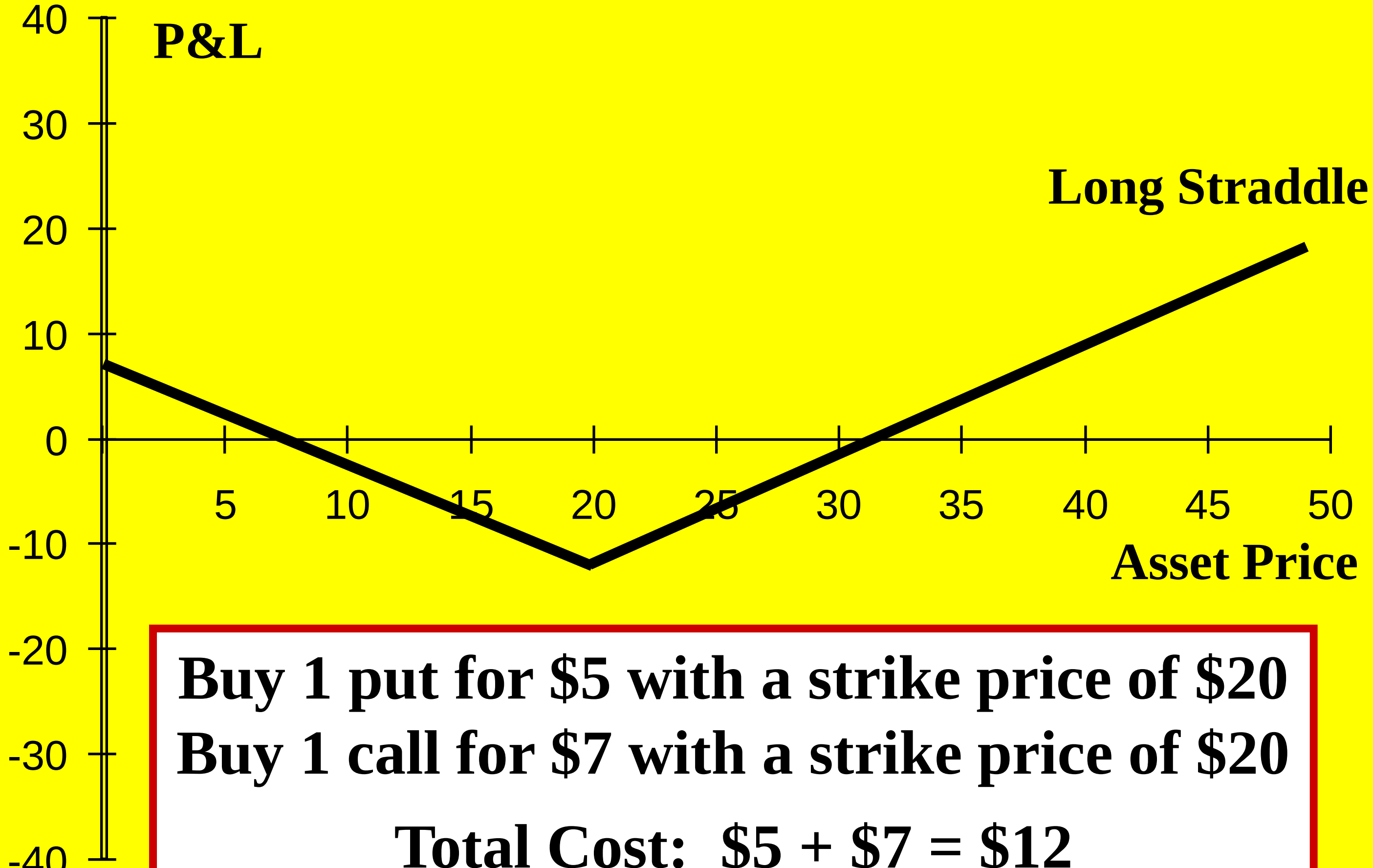
- Cost is the sum of the premium paid to buy the put and the call

Payoff

- **Max Loss:** The cost of the straddle
- **Max Gain:** Unlimited as asset price rises; as asset price falls, the the strike price less the cost of the straddle

Option Payoffs/Strategies

LOS 1 A a), b) c) Long Straddles (Kolb pp. 316-318)



Buy 1 put for $5 with a strike price of $20

Buy 1 call for $7 with a strike price of $20

Total Cost: $5 + $7 = $12

Option Payoffs/Strategies

LOS 1 A a), b) c) Short Straddles (Kolb pp. 316-318)

Definition

- A **Short Straddle** consists of a short put and a short call, both with the same strike price and expiration.
- Used generally as a bet on volatility rather than on direction

Cost Paid/Premium Received

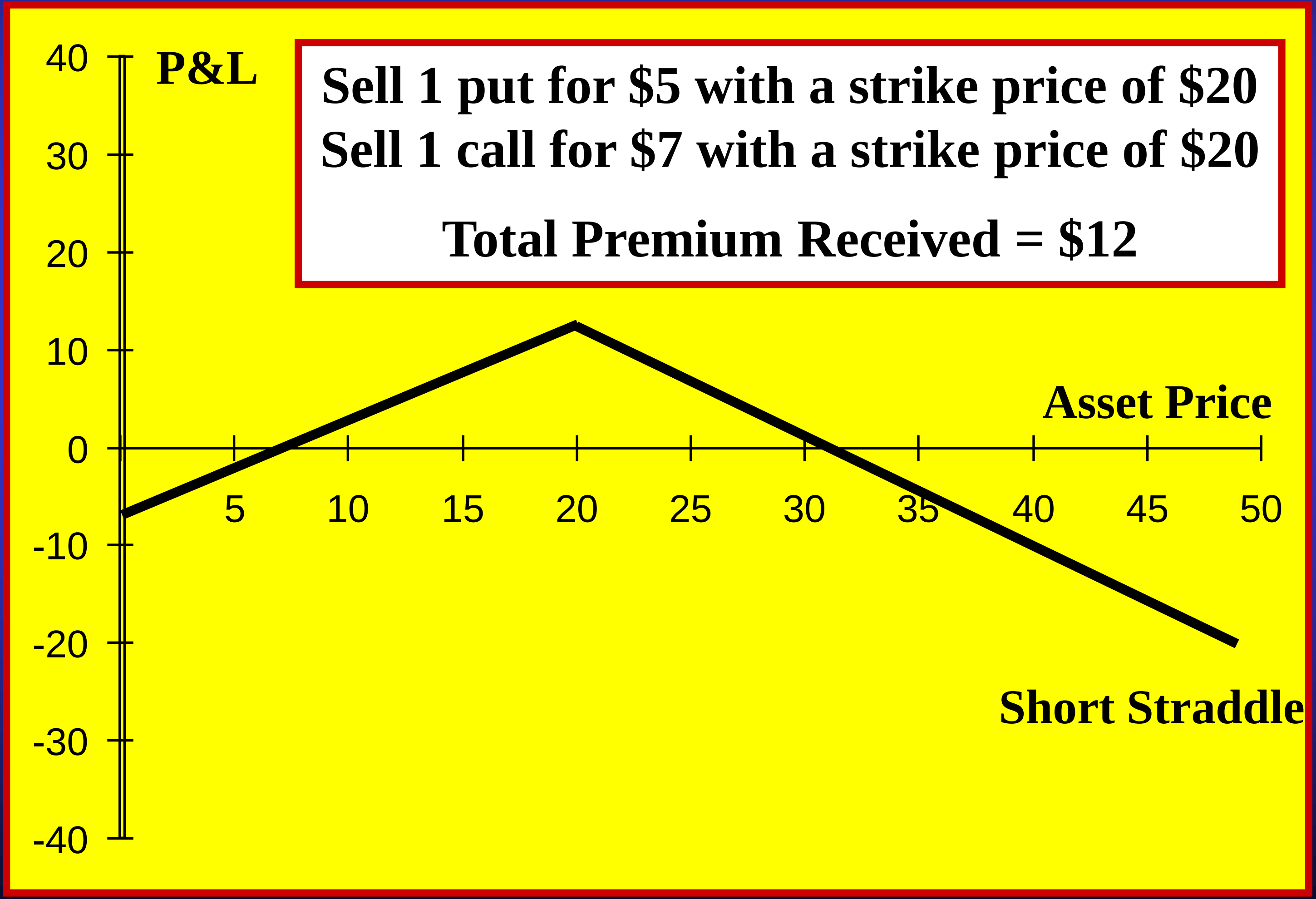
- Premium received is the sum of the prices on the put and the call

Payoff

- **Max Gain:** The premium received.
- **Max Loss:** Unlimited as asset price rises; as asset price falls, the strike price less the premium received on the strangle

Option Payoffs/Strategies

LOS 1 A a), b) c) Short Straddles (Kolb pp. 316-318)



Option Payoffs/Strategies

LOS 1 A a), b) c) Long Strangles (Kolb pp. 318-320)

Definition

- A **Long Strangle** consists of a long put with a lower strike price and a long call with a higher strike, both with same expiration.
- Used generally as a bet on volatility rather than on direction

Cost Paid/Premium Received

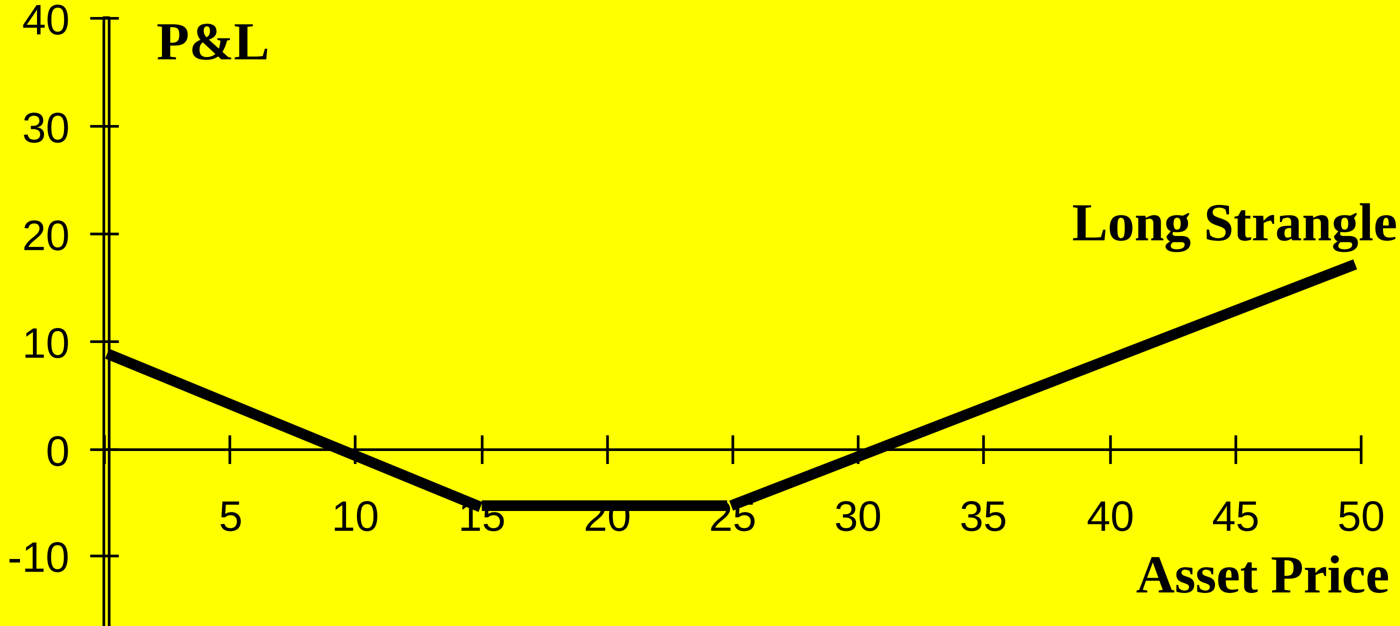
- Cost is the sum of the prices paid for the put and the call

Payoff

- **Max Gain:** Unlimited as asset price rises; as asset price falls, the put's strike price less the premium received on the strangle.
- **Max Loss:** The premium received on the strangle

Option Payoffs/Strategies

LOS 1 A a), b) c) Long Strangles (Kolb pp. 318-320)



Buy 1 put for $2 with a strike price of $15
Buy 1 call for $4 with a strike price of $25

Total Cost = $2 + $4 = $6

Option Payoffs/Strategies

LOS 1 A a), b) c) Short Strangles (Kolb pp. 318-320)

Definition

- A **Short Strangle** consists of a short put with a lower strike price and a short call with a higher strike, both with same expiration.
- Used generally as a bet on volatility rather than on direction

Cost Paid/Premium Received

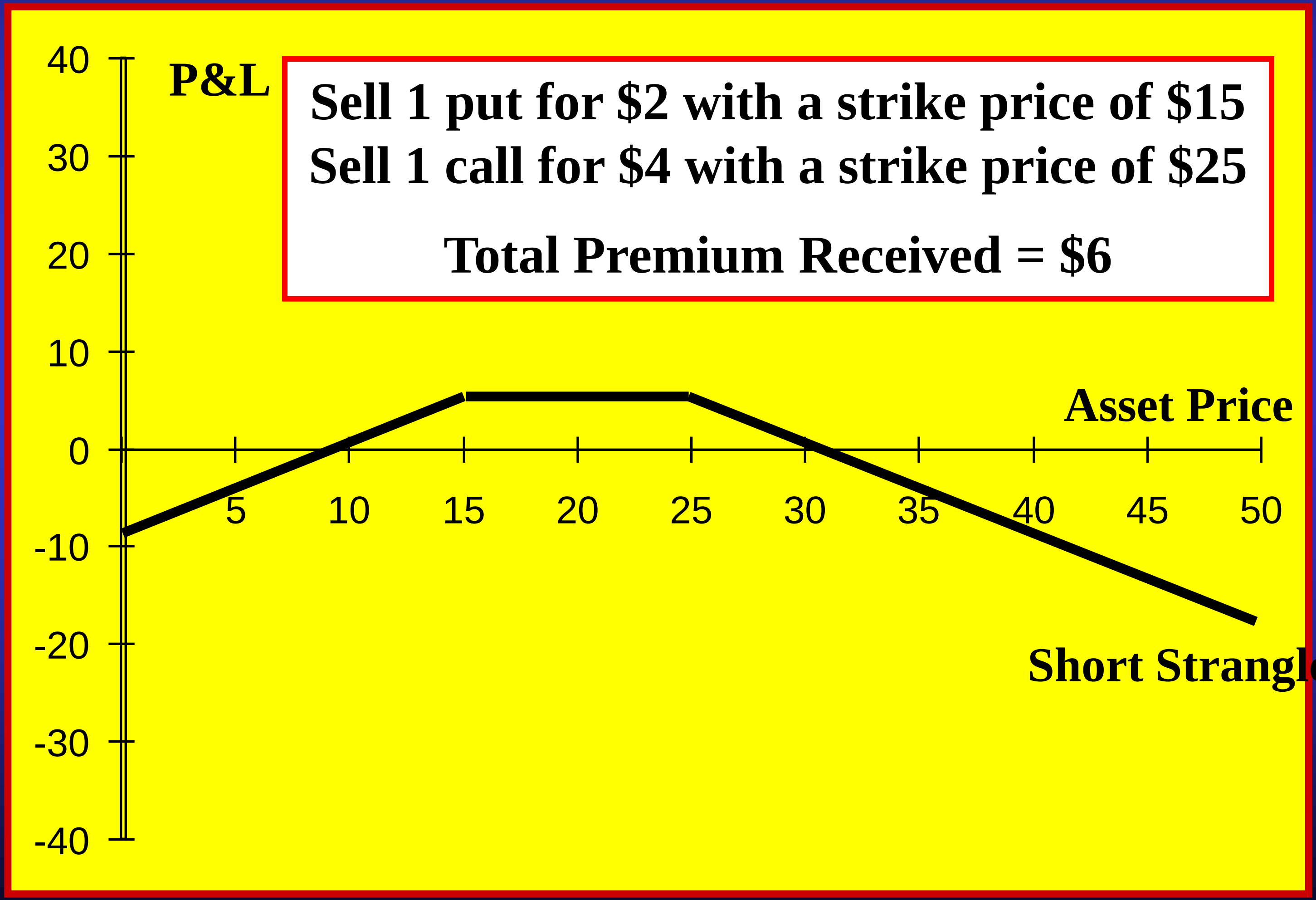
- Premium received is the sum of the prices on the put and the call

Payoff

- **Max Gain:** The premium received on the strangle.
- **Max Loss:** Unlimited as asset price rises; as asset price falls, the put’s strike price less the premium received on the strangle

Option Payoffs/Strategies

LOS 1 A a), b) c) Short Strangles (Kolb pp. 318-320)



Option Payoffs/Strategies

LOS 1 A a), b) c) Option Spreads (Kolb pp. 320-323)

Definition (reverse for Bear Spread)

- A **Call Bull Spread** consists of a long position in a call with a lower strike price and a short position in a call with a higher strike price, both with the same expiration date and underlying asset.
- Used generally as a controlled directional bet

Cost

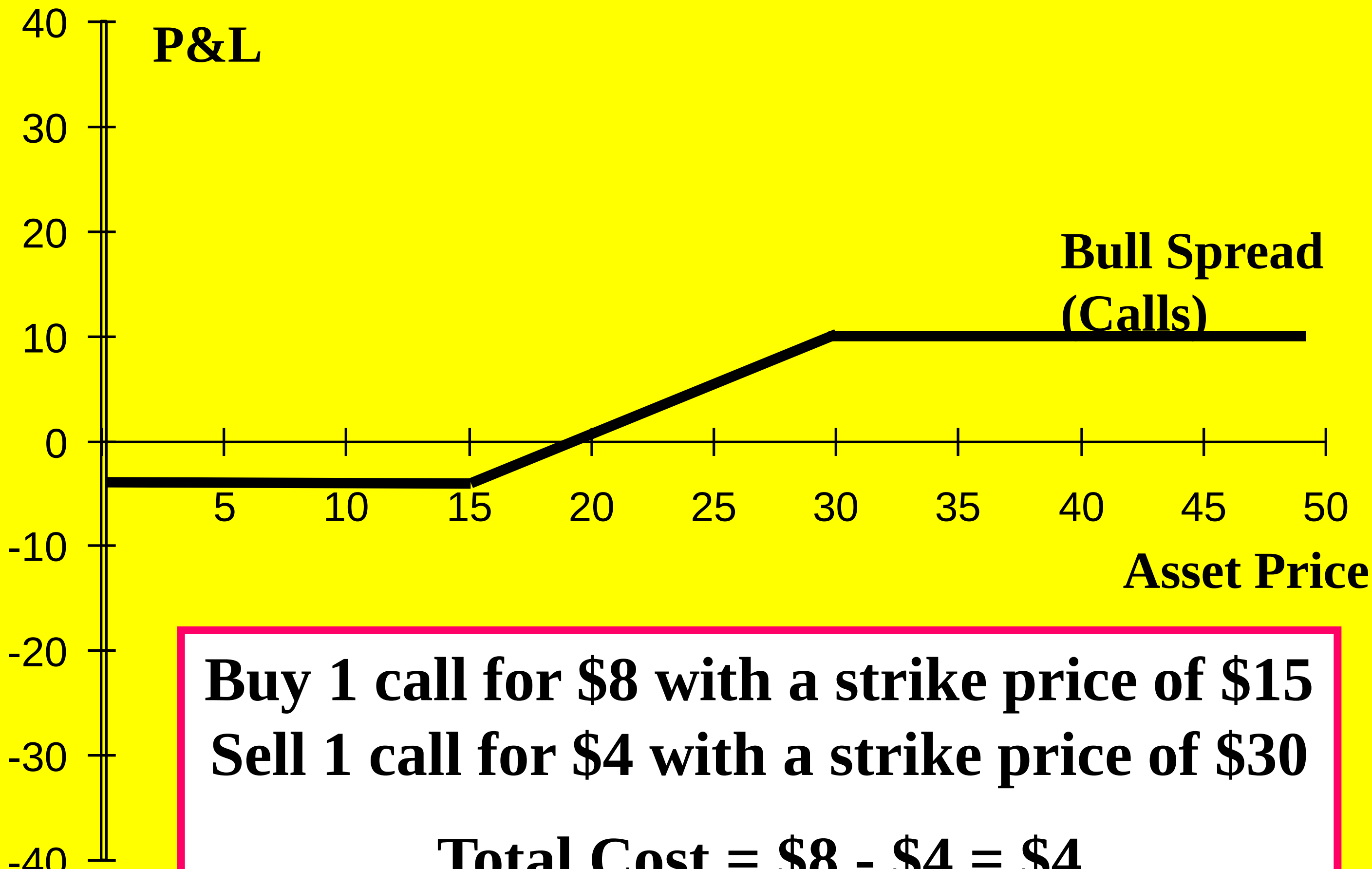
- The net of the premium paid on the long call less the premium received on the short call

Payoff to the Bull Spread (Reverse for Bear Spread)

- **Max Loss:** Cost of the Spread
- **Max Gain:** Strike price differential less the cost of the Spread

Option Payoffs/Strategies

LOS 1 A a), b) c) Option Spreads (Kolb pp. 320-323)



Buy 1 call for $8 with a strike price of $15

Sell 1 call for $4 with a strike price of $30

Total Cost = $8 - $4 = $4

Option Payoffs/Strategies

LOS 1 A a), b) c) Option Spreads (Kolb pp. 320-323)

Option Payoffs/Strategies

LOS 1 A a), b) c) Option Spreads (Kolb pp. 320-324)

	BULL SPREAD		BEAR SPREAD	
	With Calls	**With Puts**	**With Calls**	**With Puts**
Lower Strike	Long Call	Long Put	Short Call	Short Put
Upper Strike	Short Call	Short Put	Long Call	Long Put
Net Cash Flow	Pay a Cost	Receive a Premium	Receive a Premium	Pay a Cost
Max Loss	Premium Paid	Strike Diff Less Prem	Strike Diff Less Cost	Premium Paid
Max Gain	Strike Diff Less Cost	Premium Received	Premium Received	Strike Diff Less Prem

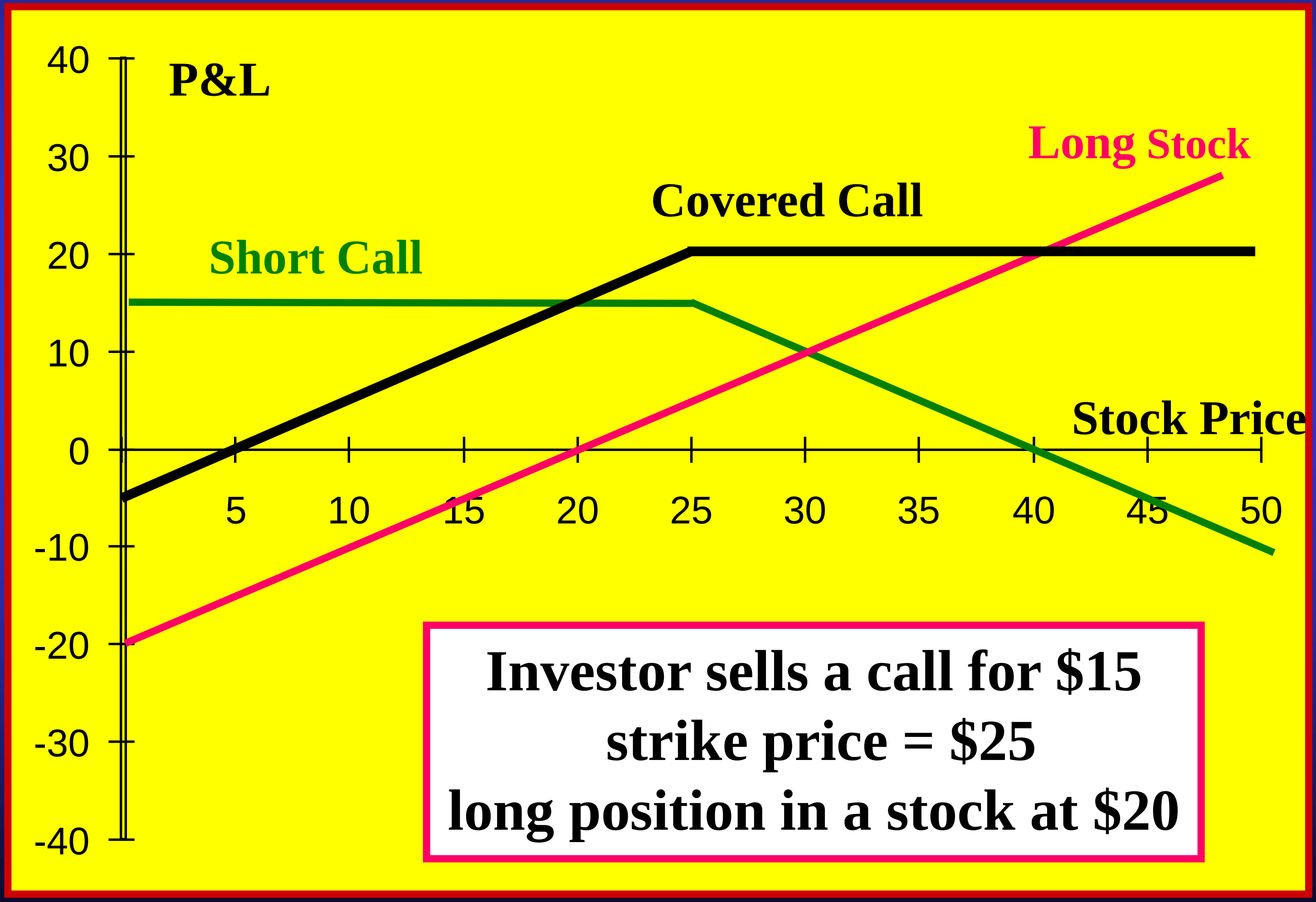
Option Payoffs/Strategies

LOS 1 A a), b) c) Option Combos (Kolb pp. 324-335)

OPTION COMBO	WHEN TO USE IT	HOW TO FORM A LONG POSITION
Box Spread	Earn/pay the risk free rate; arbitrage	Bull spread in calls Bear spread in puts
Butterfly Spread	Take a controlled volatility bet	2 long calls diff strike 2 short calls same strike
Condor Spread	Take a controlled range volatility bet	Bull spread w/ 2 strikes Bear spread, higher strikes
Ratio Spread	Directional bet with varying exposure	Long & short options in different proportions
Calendar Spread	Combined time decay bet & Volat/Direct.	Any spread with an expiration mismatch

Option Payoffs/Strategies

LOS 1 A d) Covered Calls (Kolb pp. 335-336)



Option Payoffs/Strategies

LOS 1 A d) Covered Calls (Kolb pp. 335-336)

Covered Call Comments

Why is it used?

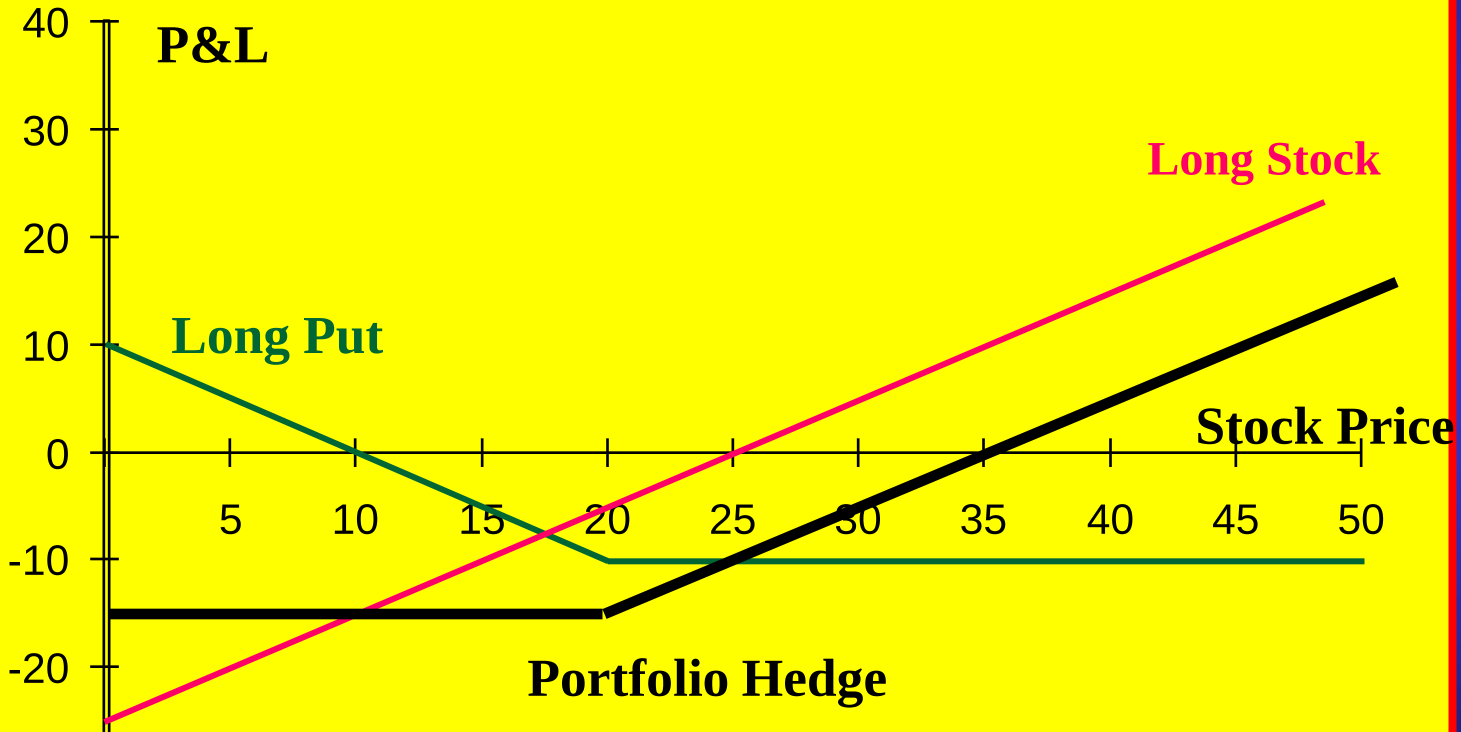
- ☞ To provide income enhancement
- ☞ To provide risk reduction

When does it work, when does it fail?

- ☞ Works when the asset does not rise to a level in excess of the sum of the strike price plus the call option premium
- ☞ Fails when the asset rises to a level in excess of the sum of the strike price plus the call option premium

Option Payoffs/Strategies

LOS 1 A d) Portfolio Hedging (Kolb pp. 337-338)



buy a put for $10
strike price = $20
hold a long position in a stock at $25

Option Payoffs/Strategies

LOS 1 A d) Portfolio Hedging (Kolb pp. 337-338)

Portfolio Hedging(Protective Put)

Why is it used?

☞ To provide a floor below which the asset value won’t fall

☞ To provide risk reduction to an overall portfolio

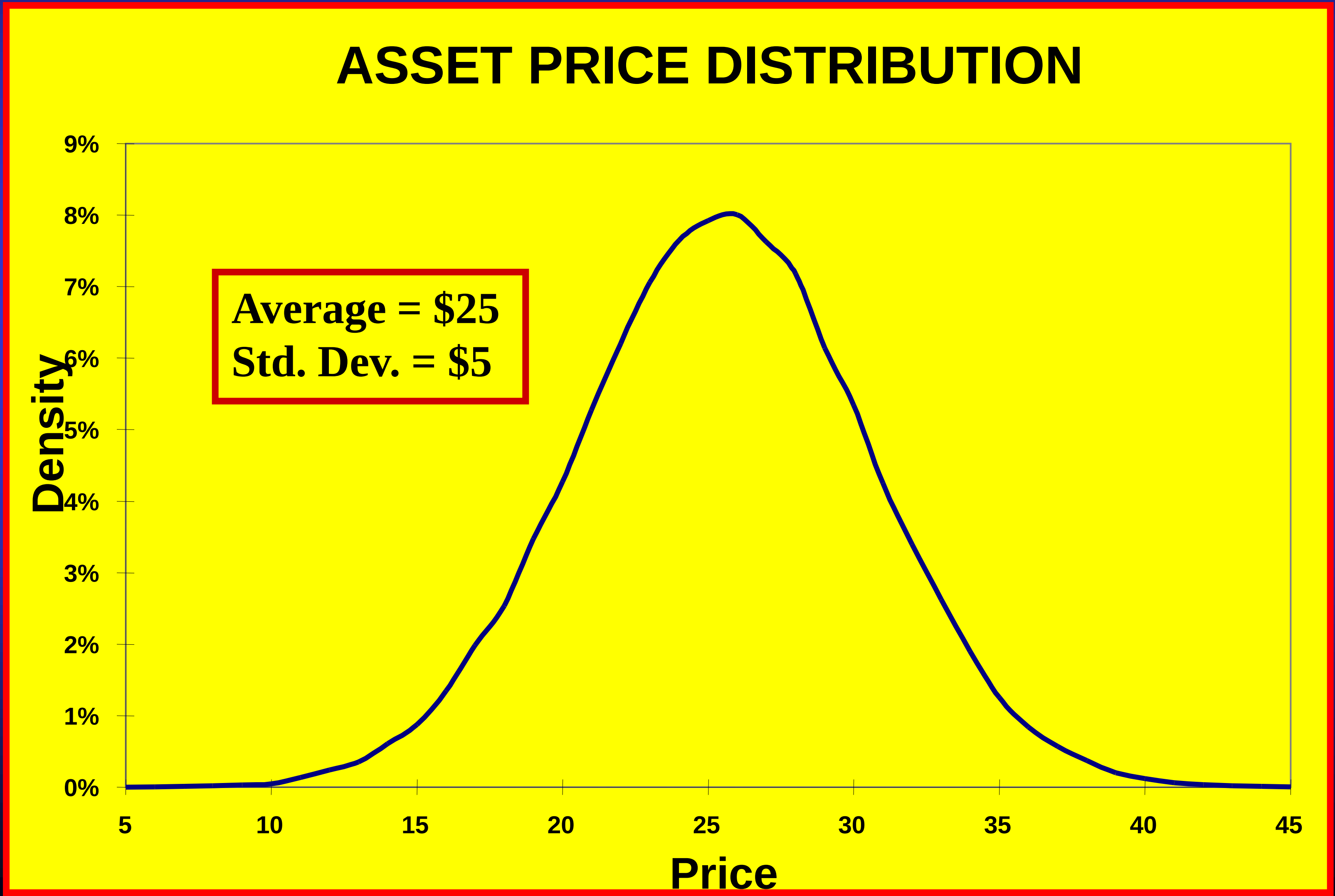
When does it work, when does it fail?

☞ Works when the asset falls to a level lower than the strike price less the put option premium

☞ Fails when the asset price stays above the strike price less the put option premium

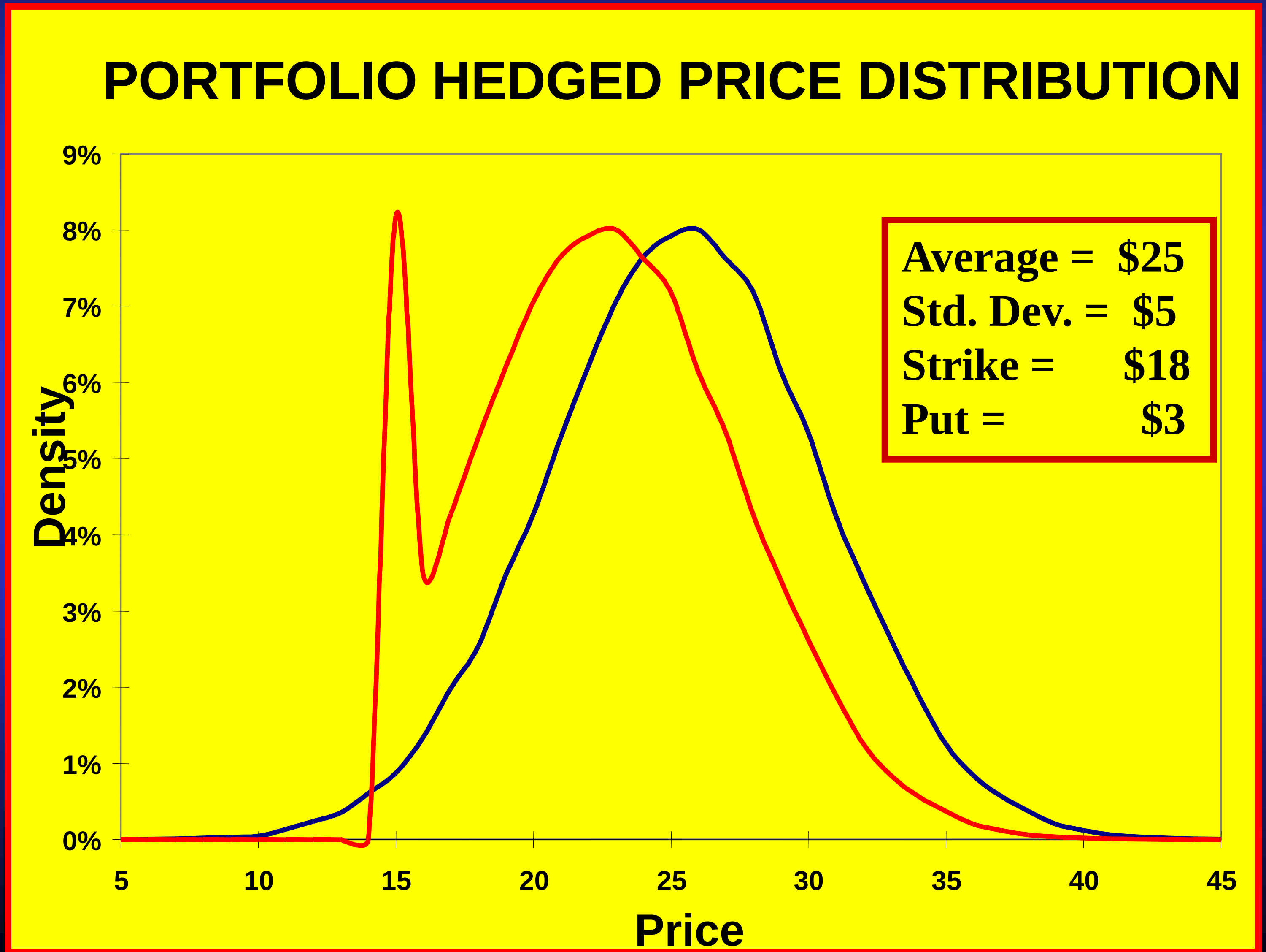
Option Payoffs/Strategies

LOS 1 A d) Portfolio Hedging (Kolb pp. 339-341)



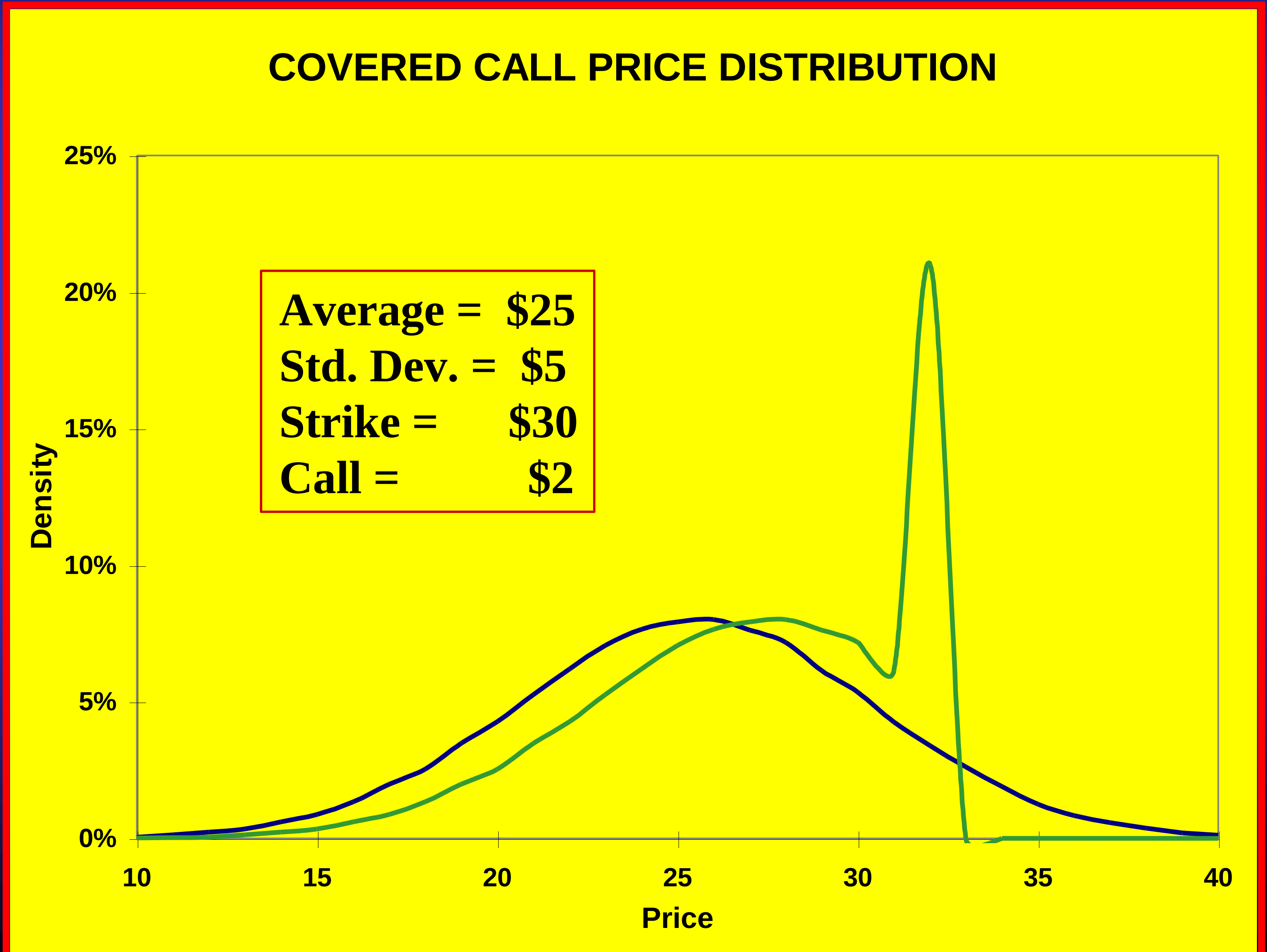
Option Payoffs/Strategies

LOS 1 A d) Hedged Asset Distribution (Kolb pp. 339-341)



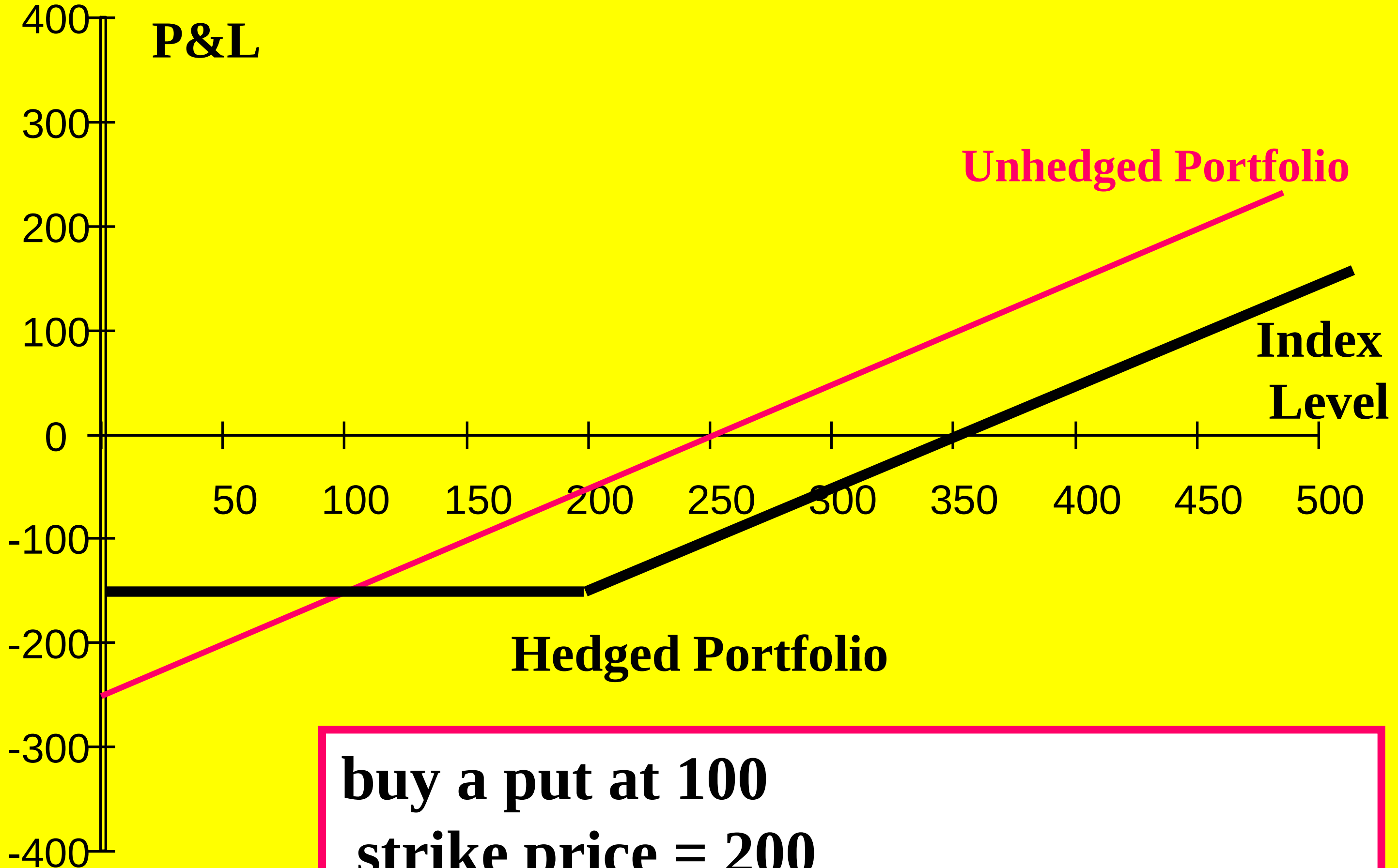
Option Payoffs/Strategies

LOS 1A d) Covered Asset Distribution (Kolb pp. 339-341)



Option Payoffs/Strategies

LOS 1 A i) Portfolio Insurance (Kolb pp. 337-339)



buy a put at 100
strike price = 200
hold a portfolio similar to index at 250

Option Payoffs/Strategies

LOS 1A i) Portfolio Insurance (Kolb pp. 337-339)

Portfolio Insurance vs. Portfolio Hedging

Portfolio Insurance is the act of buying an index put to form a protective index floor for the overall value of the portfolio.

Portfolio Hedging is the act of establishing one or more protective puts on the individual stock holdings within a portfolio by buying puts on one or more stocks held in the portfolio.

Option Payoffs/Strategies

LOS 1 A f) Put-Call Parity (Kolb pp. 345-346)

Formula: $c_t - p_t = S_t - e^{-rt}X$

Definitions

☞ c_t = theoretical value of a call with time "t" to expiration, a strike price of X, and with an underlying asset valued at S.

☞ p_t = theoretical value of a put with time "t" to expiration, a strike price of X, and with an underlying asset valued at S.

☞ X = strike price of the put and call

☞ r = the risk free rate over time period "t"

☞ S_t = the value of the asset underlying both the put and call

Option Payoffs/Strategies

LOS 1 A f), h) Put-Call Parity (Kolb pp. 345-346)

<u>The Real World Holds to Put-Call Parity Because:</u>

☞**Any Instrument Can be Synthesized** - Holding any three items in the Put-Call Parity equation will "synthesize" the fourth

☞**Synthetics Have the Same Flows** - Synthetics have the same cash flows as their real world equivalents under all circumstances

☞ **No Free Lunch** - If Put/Call Parity is not observed you could earn riskless profits creating a long synthetic and selling the real world instrument (or vice versa)

☞ **Early Exercise** - Put/Call parity holds for European options but may not hold for American options due to early exercise.

Option Payoffs/Strategies

LOS 1 A e) Creating Synthetic Stock (Kolb pp. 342-345)

Formula: $S_t = c_t - p_t + e^{-rt}X$

<u>Construction of Synthetic Stock (with strike X, until time t):</u>

- ☞ Buy 1 call with an at-the-money strike ($X = S_t$)
- ☞ Lend the discounted value of X ($= S_t$) until time t at rate r
- ☞ Sell 1 put option with an at-the-money strike ($X = S_t$)

<u>Behavior of the Synthetic Stock at Expiration (Time T):</u>

- ☞ You receive the full value of the loan for X ($= S_t$)
- ☞ If $S_T > X$, exercise call, buy stock for S_t that's worth S_T
- ☞ If $S_T < X$, call = 0, forced to buy stock from put for S_t

Option Payoffs/Strategies

LOS 1 A g) Creating Synthetic Calls (Kolb pp. 345-346)

Formula: $c_t = p_t + S_t - e^{-rt}X$

Construction of a Synthetic Call (with strike X, expiration t):

- ☞ Borrow the discounted value of X until time t at rate r
- ☞ Buy 1 unit of the underlying asset at S_t
- ☞ Buy 1 put option with strike X, expiration t

Behavior of the Synthetic Call at Expiration (Time T):

- ☞ You must pay the full value of the loan for X
- ☞ If $S_T > X$, put is worthless, pay off loan, keep $S_T - X$
- ☞ If $S_T < X$, sell stock to put seller for X, pay loan, keep \$0

Option Payoffs/Strategies

LOS 1 A g) Creating Synthetic Puts (Kolb pp. 345-346)

Formula: $p_t = c_t - S_t + Xe^{-rt}$

<u>Construction of a Synthetic Put (with strike X, expiration t):</u>

- ☞ Sell 1 unit of the underlying asset S_t
- ☞ Lend the discounted value of X until time t at rate r
- ☞ Buy 1 call option with strike X, expiration t

<u>Behavior of the Synthetic Put at Expiration (Time T):</u>

- ☞ You receive the full value of the loan for X
- ☞ If $S_T > X$, exercise call, buy stock with X, keep $0
- ☞ If $S_T < X$, call is worthless, pay S for stock, keep $X - S_T$

Option Payoffs/Strategies

LOS 1A j) Creating Synthetics Overall (Kolb pp. 341-346)

Synthetic Securities and Put/Call Parity

Put/Call Parity Theorem contains four items:

(1) Put
(2) Call
(3) Bond
(4) Stock

Any of these four instruments can be "synthesized" by:

- Isolating it on the left-hand side of the put/call theorem

- Holding the remaining three items as indicated

EUROPEAN OPTION PRICING

Option Pricing

LOS 1 B a) Option Replication (Kolb p. 382)

The Secret To Option Pricing: Replication

The key insight, put forth by Black and Scholes in 1973 (but fundamental to every option pricing model ever developed) is:

Assuming the existence of a riskless interest rate and a model of how the underlying asset price moves, the theoretical value of a simple European call option with a fixed strike price should equal a the value of a definable pattern of borrowing and lending combined with holding the asset long or short.

Option Pricing

LOS 1 B c) Single Period Binomial (Kolb pp. 382-385)

Single Period Binomial Pricing Model Assumptions

☞ **Single Period -** A European call option with 1 time period to expiration

☞ **Binomial Price Moves -** The underlying asset can only move up by a fixed percentage or down by a fixed percentage until expiration.

☞ **Riskless Bond -** An investor can borrow or lend for the single period at a given riskless rate of interest.

☞ **Portfolio Creation -** An investor can hold any portfolio combination of the underlying asset and the bond (long or short)

Option Pricing

LOS 1 B c), d) Single Period Binomial (Kolb pp. 382-385)

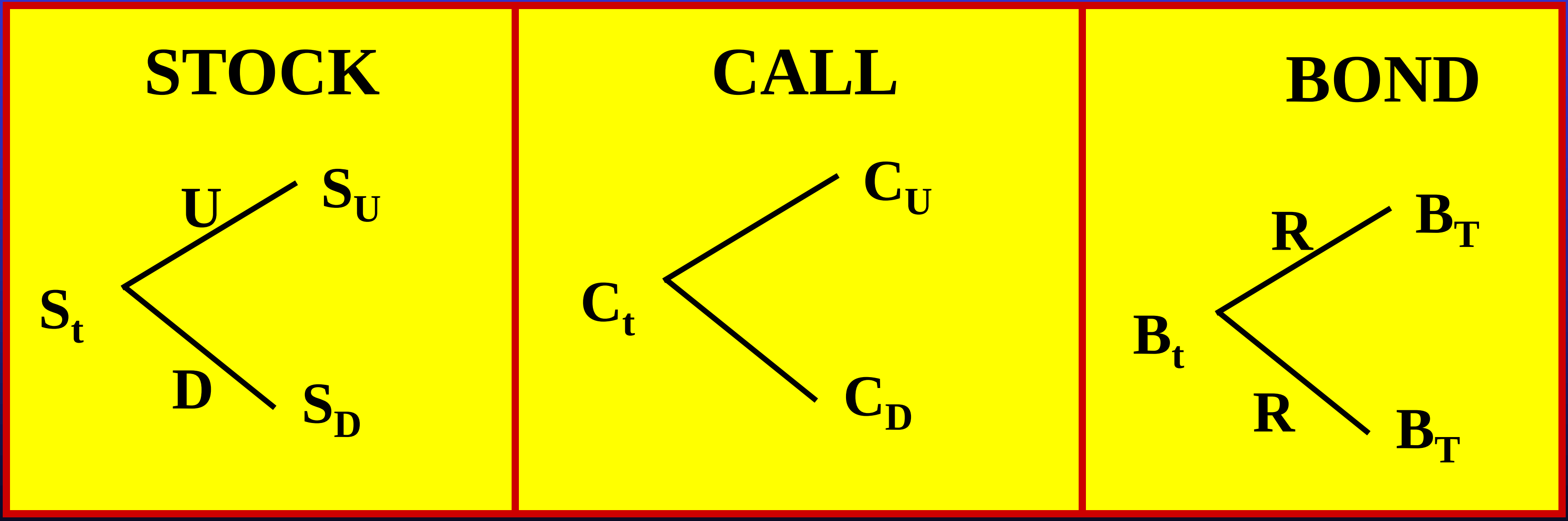
Let:

U = 1 + percentage increase in the stock price

D = 1 - percentage decrease in the stock price

R = 1 + the riskless one period interest rate

C_t = the current value of a call option



Option Pricing

LOS 1 B c) Single Period Binomial (Kolb pp. 382-385)

MATHEMATICAL FORMULAS:

$$B_t^* = \frac{C_u D - C_D U}{(U - D)R}$$

$$N^* = \frac{C_u - C_D}{(U - D)S_t}$$

Which Leads To:

$$C_t^* = N^* S_t - B_t^*$$

Option Pricing

LOS 1 B c) Single Period Binomial (Kolb pp. 382-385)

A Single Period Binomial Option Pricing Example

☞ **Stock Price -** The underlying asset is currently selling at $50

☞ **Binomial Price Moves -** The underlying asset can move up by 10% or down by 8% over the one period til expiration

☞ **Riskless Bond -** An investor can borrow or lend for the one period at a 5% riskless interest rate

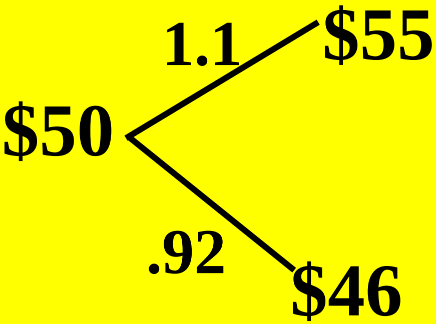
☞ **Call Option -** A European call option is available for one period with a strike price of $45.

WHAT IS THE CALL OPTION WORTH?

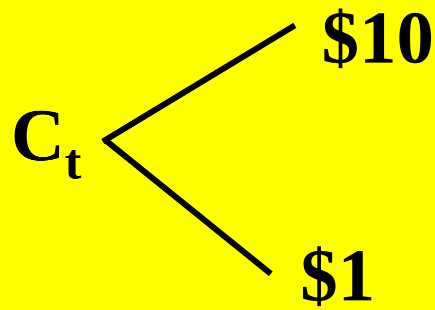
Option Pricing

LOS 1 B c), d) Single Period Binomial (Kolb pp. 382-385)

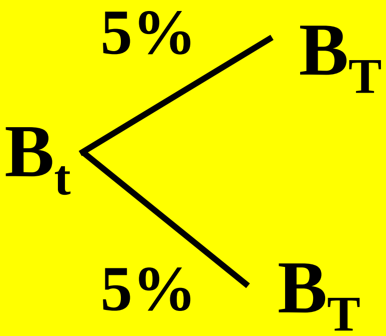
STOCK



CALL



BOND



$$N^* = \frac{C_U - C_D}{(U-D)S_t} = \frac{\$10 - \$1}{(1.1-.92)\text{x}50} = 1.0$$

Option Pricing

LOS 1 B c) Single Period Binomial (Kolb pp. 382-385)

$$B_t^* = \frac{C_U D - C_D U}{(U-D)R} = \frac{\$10 \text{ x } .92 - \$1 \text{ x } 1.1}{(1.1 - .92) \text{ x } 1.05} = \$42.857$$

Which Leads To:

$$C_t^* = N^* S_t - B_t^* = 1.0 \text{ x } \$50 - \$42.857$$
$$= \$7.142$$

Option Pricing

LOS 1 B c) Two Period Binomial (Kolb pp. 385-386)

Two Period Binomial Pricing Model Assumptions

☞ **Two Periods -** Assume a European call option contract exists with two distinct time periods from now (time of purchase) to expiration

☞ **Binomial Price Moves -** The underlying asset can only move up by a fixed percentage or down by a fixed percentage each period.

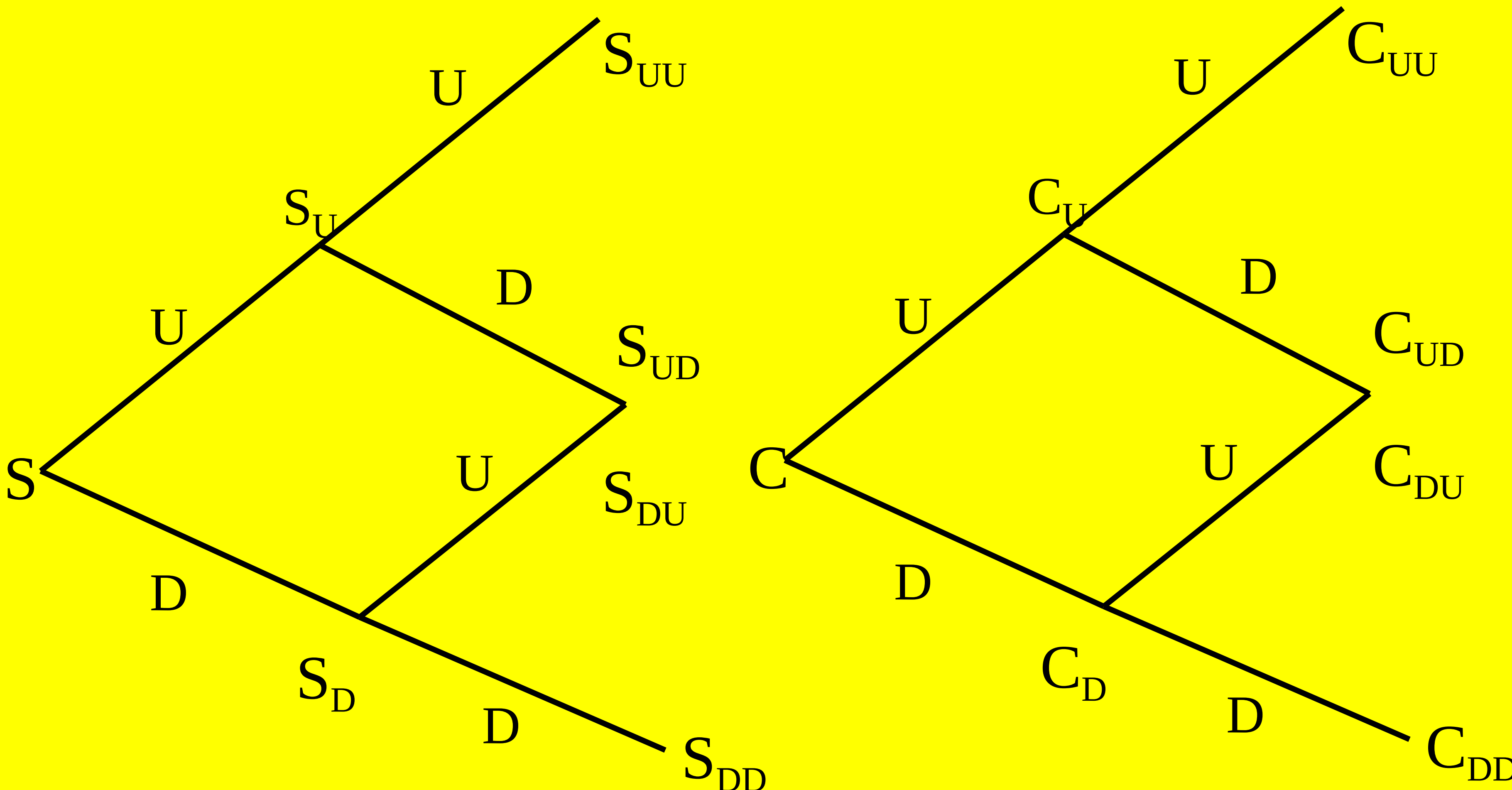
☞ **Riskless Bond -** An investor can borrow or lend for the two periods at a given riskless rate of interest.

☞ **Independence -** The probability of a price move (up or down) in the first time period is independent from the probability of a price move (up or down) in the second time period

Option Pricing

LOS 1 B c), d) Two Period Binomial (Kolb pp. 385-386)

A Two-Period Binomial Option Pricing Model



Option Pricing

LOS 1 B c) Single Period Binomial (Kolb pp. 385-386)

MATHEMATICAL FORMULAS:

Π_U = Probability of an up price move in a single period

Π_D = Probability of a down price move in a single period

Π_{UU} = Probability of two up moves in two periods = $(\Pi_U)(\Pi_U)$

Π_{DD} = Probability of two down moves in two periods = $(\Pi_D)(\Pi_D)$

Π_{UD} = Probability of one up, one down move in two periods

$= (\Pi_U)(\Pi_D) = (\Pi_U)(\Pi_D) = \Pi_{DU}$

Option Pricing

LOS 1 B c) Two Period Binomial (Kolb pp. 385-386)

MATHEMATICAL FORMULAS (cont'd):

$$C_t = \frac{\pi_{UU} C_{UU} + \pi_{UD} C_{UD} + \pi_{DU} C_{DU} + \pi_{DD} C_{DD}}{R^2}$$

Option Pricing

LOS 1 B c) Two Period Binomial (Kolb pp. 385-386)

A Two Period Binomial Option Pricing Example

☞ **Stock Price -** The underlying asset is currently selling at $75

☞ **Binomial Price Moves -** The underlying asset can move up by 12% (with probability 40%) or down by 7% (with probability 60%) during either period

☞ **Riskless Bond -** An investor can borrow or lend across both periods at a 5% riskless interest rate per period

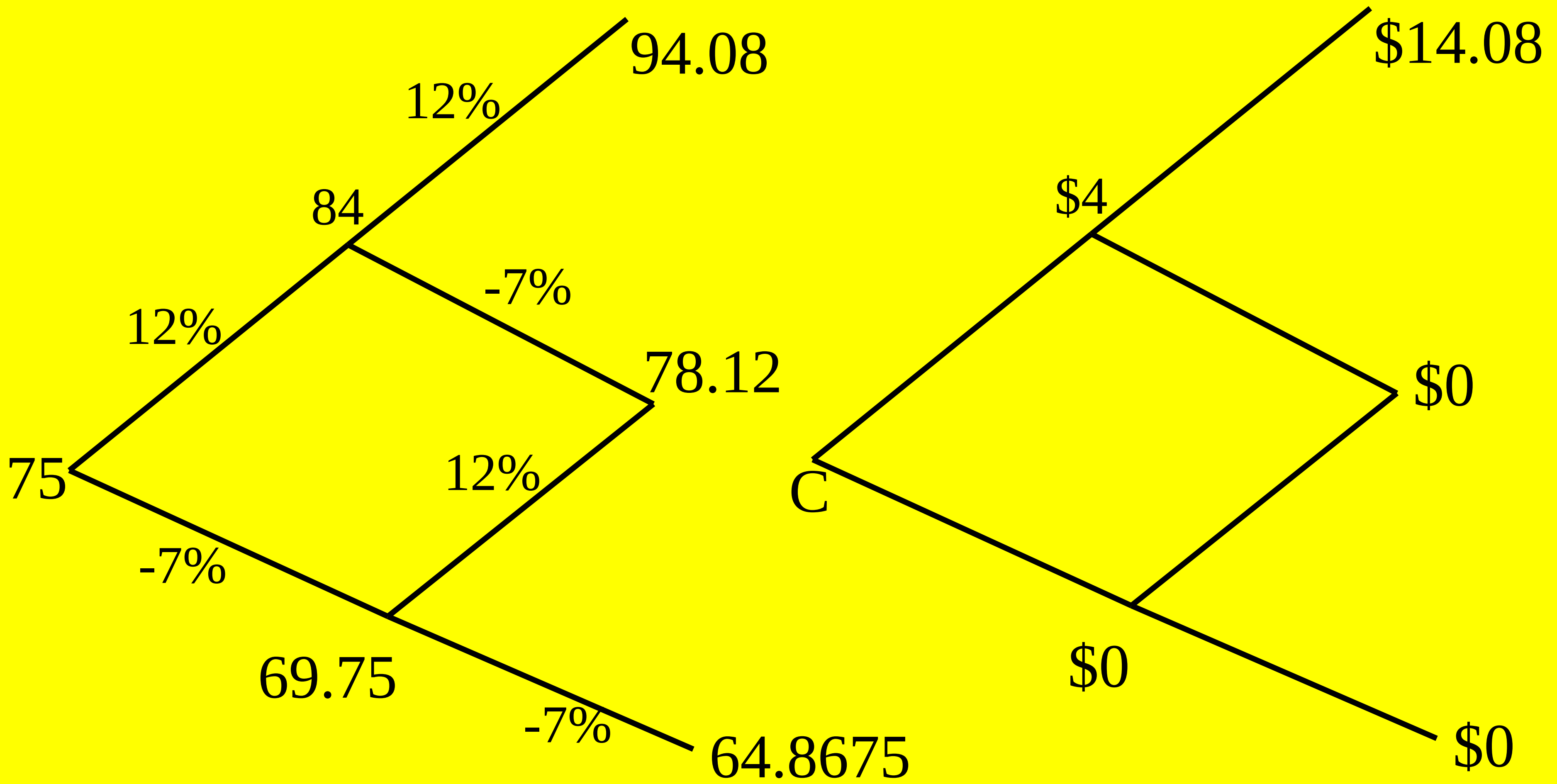
☞ **Call Option -** A European call option is available for two periods with a strike price of $80.

WHAT IS THE CALL OPTION WORTH?

Option Pricing

LOS 1 B c), d) Two Period Binomial (Kolb pp. 385-386)

A Two-Period Binomial Option Pricing Model



Option Pricing

LOS 1 B c) Two Period Binomial (Kolb pp. 385-386)

Two Period Example: Answer

$$C_t = \frac{(0.4)(0.4)(\$14.08) + (2)(0.4)(0.6)(\$0) + (0.6)(0.6)(\$0)}{(1.05)^2}$$

$$= \frac{\$2.2528}{1.1025} = \$2.0434$$

Option Pricing

LOS 1 B c) Black-Scholes Model (Kolb pp. 399-402)

<u>Black-Scholes Option Pricing Model: Inputs</u>

☞ **Asset Price (S) -** The current market price of the underlying asset

☞ **Time to Expiration (t) -** The amount of time, expressed similarly to the interest rate and volatility, until the option contract expires

☞ **Volatility (σ) -** The constant volatility of the assumed process that drive the underlying asset's price return

☞ **Risk Free Rate (r) -** The riskless interest rate expected to prevail until the option contract expires

☞ **Exercise Price (X) -** The price of the underlying asset at which the option buyer can force the option seller to transact

Option Pricing

LOS 1 B c) Black-Scholes Model (Kolb pp. 396-397)

Black-Scholes Option Pricing Model: Formulas

$$C_t = S_t N(d_1) - X e^{-r(T-t)} N(d_2)$$

$where:$

$$N(\cdot) = Cumulative\ Normal\ Distribution\ Function$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + (r + .5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Option Pricing

LOS 1 B c) Black-Scholes Model (Kolb pp. 399-402)

Black-Scholes Option Pricing Model: Example

You are looking at buying a 6 month European call option with a $30 exercise price. Current short-term interest rates are 6%, the underlying stock is selling at $37.50, and the annualized volatility of the stock's returns is 25%.

What should you expect to pay for this European call option under a no-arbitrage pricing scenario?

Option Pricing

LOS 1 B c) Black-Scholes Model (Kolb pp. 399-402)

Black-Scholes Option Pricing Model: Answer

First, identify from the problem the appropriate inputs to the Black-Scholes formula:

S_t = $37.50   r = 6%X = $30.00   s = 25%   (T-t) = 0.5

Second, plug the numbers into the formula, starting with d_1 and d_2:

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + .5\sigma^2\right)\left(T - t\right)}{\sigma\sqrt{T-t}}$$

$$= \frac{\ln\left(\frac{37.50}{30}\right) + \left(0.06 + 0.5 \times 0.25^2\right)\left(0.50\right)}{0.25 \times \sqrt{0.50}} = \frac{0.2231 + 0.0456}{0.125} = 1.52041$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = 1.5204 - 0.25 \times \sqrt{0.50} = 1.39539$$

Option Pricing

LOS 1 B c) Black-Scholes Model (Kolb pp. 399-402)

Black-Scholes Option Pricing Model: Answer

Finally, substitute the values for d_1 and d_2, along with other necessary inputs, into the main Black-Scholes equation:

$$C_t = S_t N(d_1) - Xe^{-r(T-t)} N(d_2)$$

$$= 37.50 \times N(1.52039) - 30e^{-0.06 \times 0.50} \times N(1.39539)$$

$$= 37.50 \times 0.93579 - 30 \times 0.97045 \times 0.91855$$

$$= 35.0921 - 26.7422$$

$$\approx \$8.35$$

Option Pricing

LOS 1 B e) Binomial Convergence (Kolb p. 396)

<u>Convergence of Multi-Period Binomial to Black-Scholes</u>

☞ Multi-period binomial model cuts the option's time to expiration into smaller and smaller time intervals.

☞ Binomial asset price moves become smaller and more numerous

☞ Asset price moves converge to a normal distribution

☞ The Black-Scholes model has all the same assumptions as the multi-period binomial, except for normally distributed prices

☞ Therefore, if you take the multi-period binomial model to the limit of an infinite number of infinitely small time periods, you get the Black-Scholes model.

Option Pricing

LOS 1 B b) Volatility Inputs (Kolb pp. 400-402)

Volatility as an Option Model Input

An estimate of the volatility of the underlying asset's return over the option's life is a key input to the Black-Scholes pricing model. There are two common approaches to forming a volatility estimate:

☞ **Actual Volatility -** Forecast the future volatility of the asset's returns by using the actual historical volatility of the asset's returns.

☞ **Implied Volatility -** Forecast the future volatility by finding that volatility which would cause the market price of similar options to be equal to the Black-Scholes model price

Both methods are detailed on the following slides

Option Pricing

LOS 1 B b), f) Actual Volatility (Kolb pp. 400-401)

Estimating <u>Actual</u> Volatility

Starting with a historical time series of the asset's prices:

☞ **Step One: Create the Log Price Relatives** - Take the log of the ratio of one period's price over the previous period's price

☞ **Step Two: Find the Raw Volatility** - Calculate the standard deviation of the log price relatives from Step One

☞ **Step Three: Annualize the Raw Volatility** - Multiply the raw volatility by the necessary constant to express it similarly to the other option variables that have a time dimension, such as time to expiration and the risk free rate (see next page)

Option Pricing

LOS 1 B b), f) Implied Volatility (Kolb pp. 401-402)

Estimating Implied Volatility

Take a set of options with the same expiration and underlying asset:

☞ **Step One: Collect Inputs -** Collect all option pricing variables (except volatility) for the options, including market prices

☞ **Step Two: Calculate all "Implied Volatilities" -** For each option find the level of volatility (by trial and error) that makes the estimated price equal the actual market price.

☞ **Step Three: Combine the Implied Volatilities -** Weight the implied volatilities from Step Two "appropriately" to get an estimate of the underlying asset's constant return variance.

Option Pricing

LOS 1 B g) Annualizing Volatility (Kolb p. 401, *420n*)

Annualizing Volatility

To convert from monthly, weekly, or daily volatility to annual:

Monthly: Annual Volatility = Monthly Volatility x sqrt(12)

Weekly: Annual Volatility = Weekly Volatility x sqrt(52)

Daily: Annual Volatility = Daily Volatility x sqrt(250)

Note: Daily might also use 252, 260, or even 365, depending on your day count assumptions.

Option Pricing

LOS 1 B h) Adjusting for Dividends (Kolb pp. 402-405)

<u>Merton's Adjustment for "Leakage"</u>

If the asset underlying a European option is expected to lose value continuously over the life of the option, adjust the option pricing formula for the "leakage" by substituting:

$$e^{-\delta t} S_0$$

which is the final reduced value of S_0, anywhere in the formula where you would have used S_0. Note that δ is the annualized continuous rate of leakage.

Note that Merton's model becomes the Black-Scholes model when $\delta = 0$.

Option Pricing

LOS 1 B h) Adjusting for Dividends (Kolb pp. 402-405)

<u>Applications of Merton's "Adjustment" to Dividends</u>

(1) When a <u>stock</u> pays a dividend the stock price falls by the amount of the dividend at the time the dividend is paid.

(2) When a <u>stock index</u> pays multiple dividends the index falls by the size of each dividend at the time each dividend is paid.

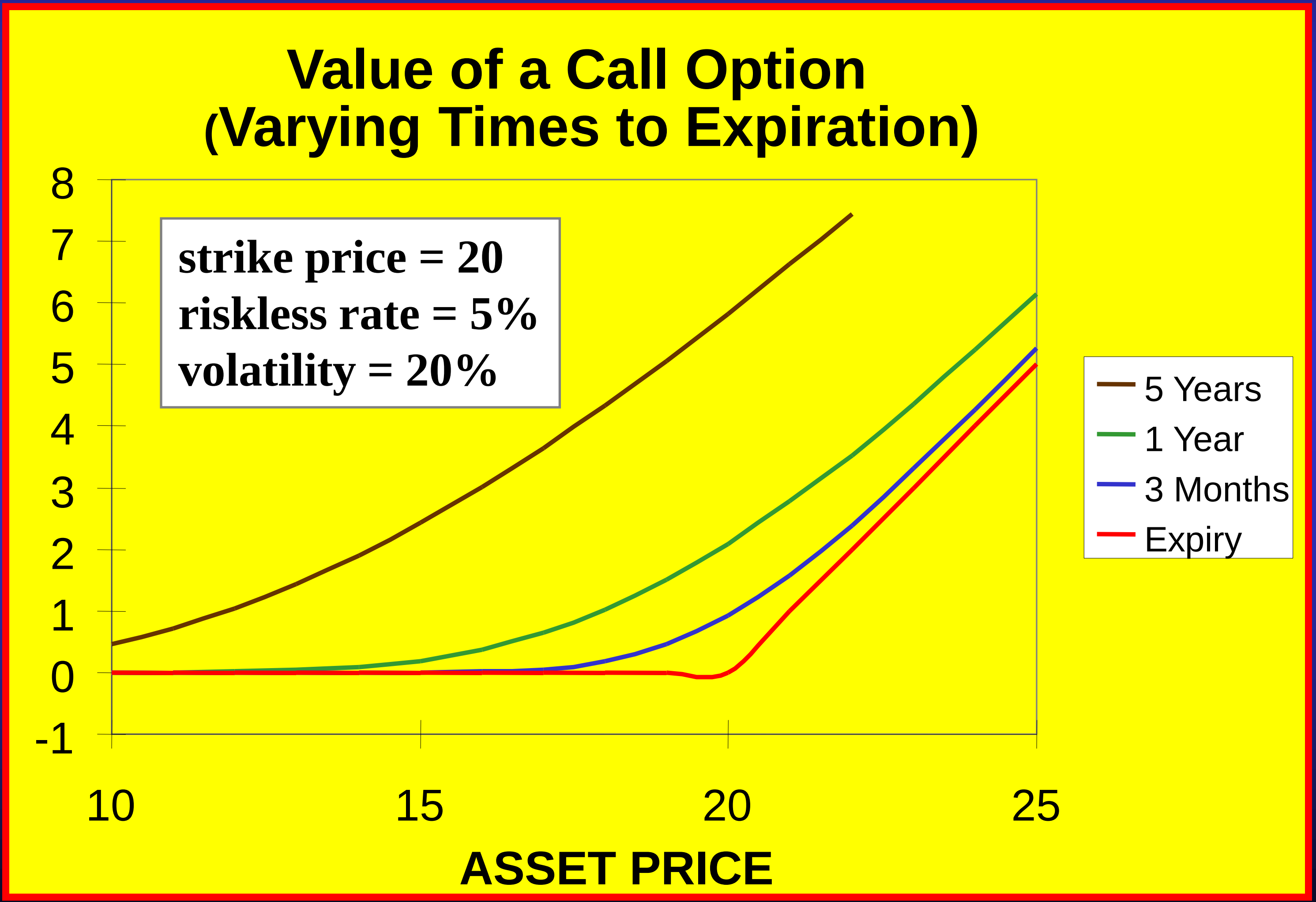
(3) If a stock index is assumed to pay a <u>continuous dividend yield</u> then the index is assumed to drop continuously at that rate, all other things equal.

THEREFORE - Merton's adjustment to the B-S formula for "leakage" applies to the continuous payment of dividends.

OPTION SENSITIVITIES AND OPTION HEDGING

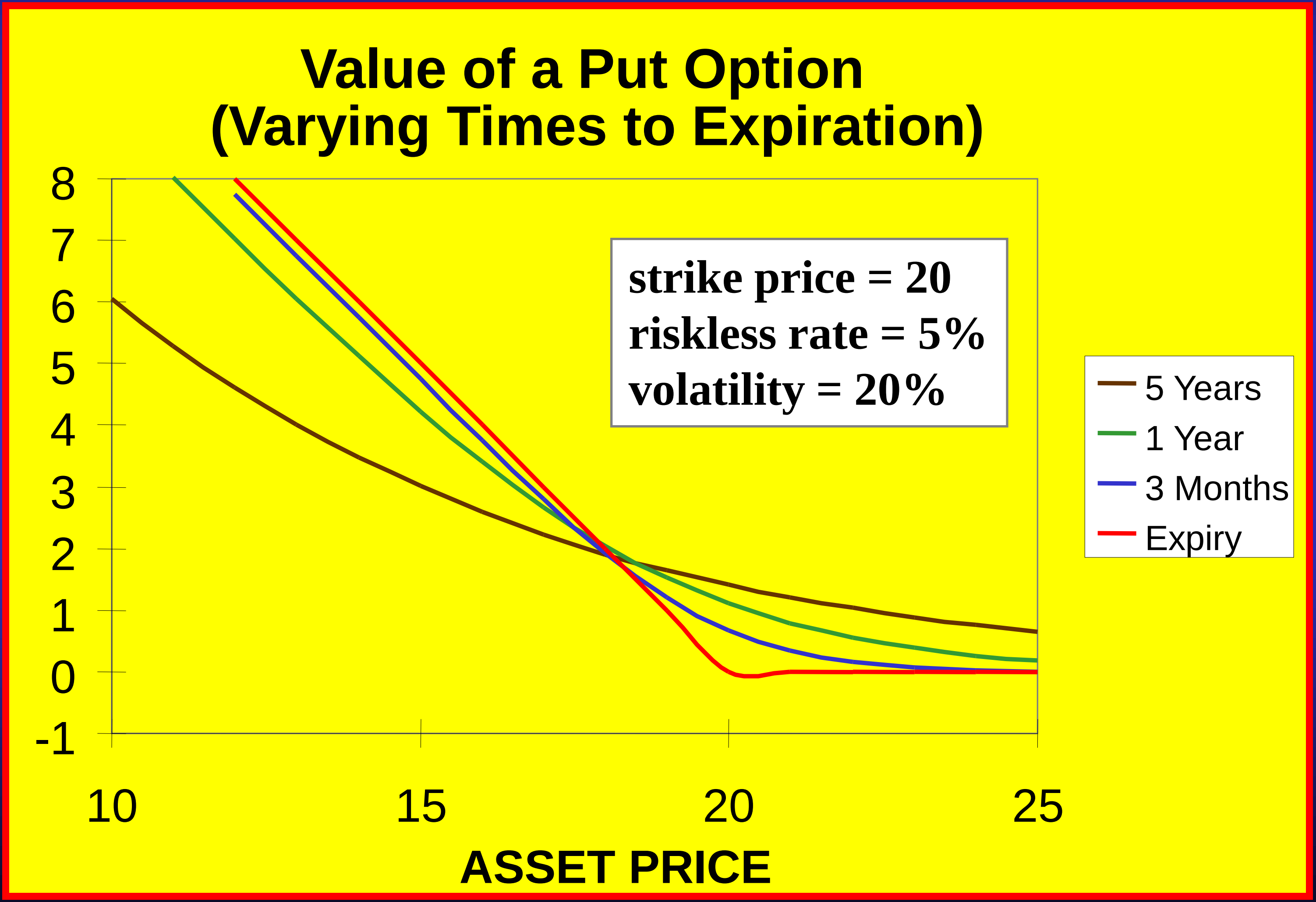
Option Sensitivities

LOS 1 C a) Pre-Expiration Payoffs (Kolb pp. 423-425)



Option Sensitivities

LOS 1 C a) Pre-Expiration Payoffs (Kolb pp. 423-425)



Option Sensitivities

LOS 1 C a) Pre-Expiration Payoffs (Kolb pp. 423-425)

Important Points

For Call Options:

☞ Longer term call options should always have higher value than shorter term call options

☞ Any call option should have a value no less than its intrinsic value

For Put Options:

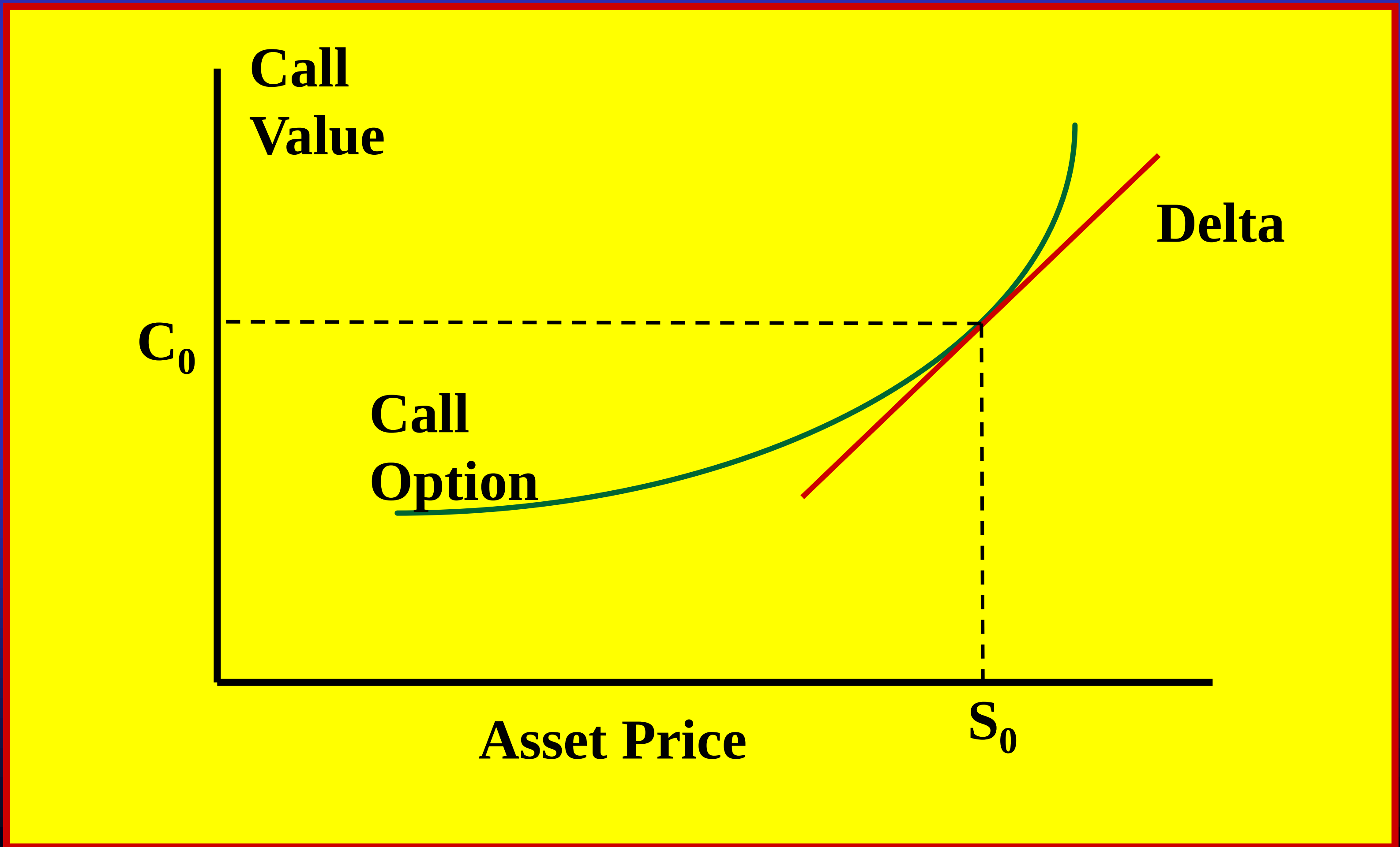
☞ Generally longer term put options have higher value than shorter term put options

☞ A very long term put option, or a put option that is very deep in the money, can be worth less than its intrinsic value

Option Sensitivities

LOS 1 C b) Call Option Delta (Kolb pp. 424-428)

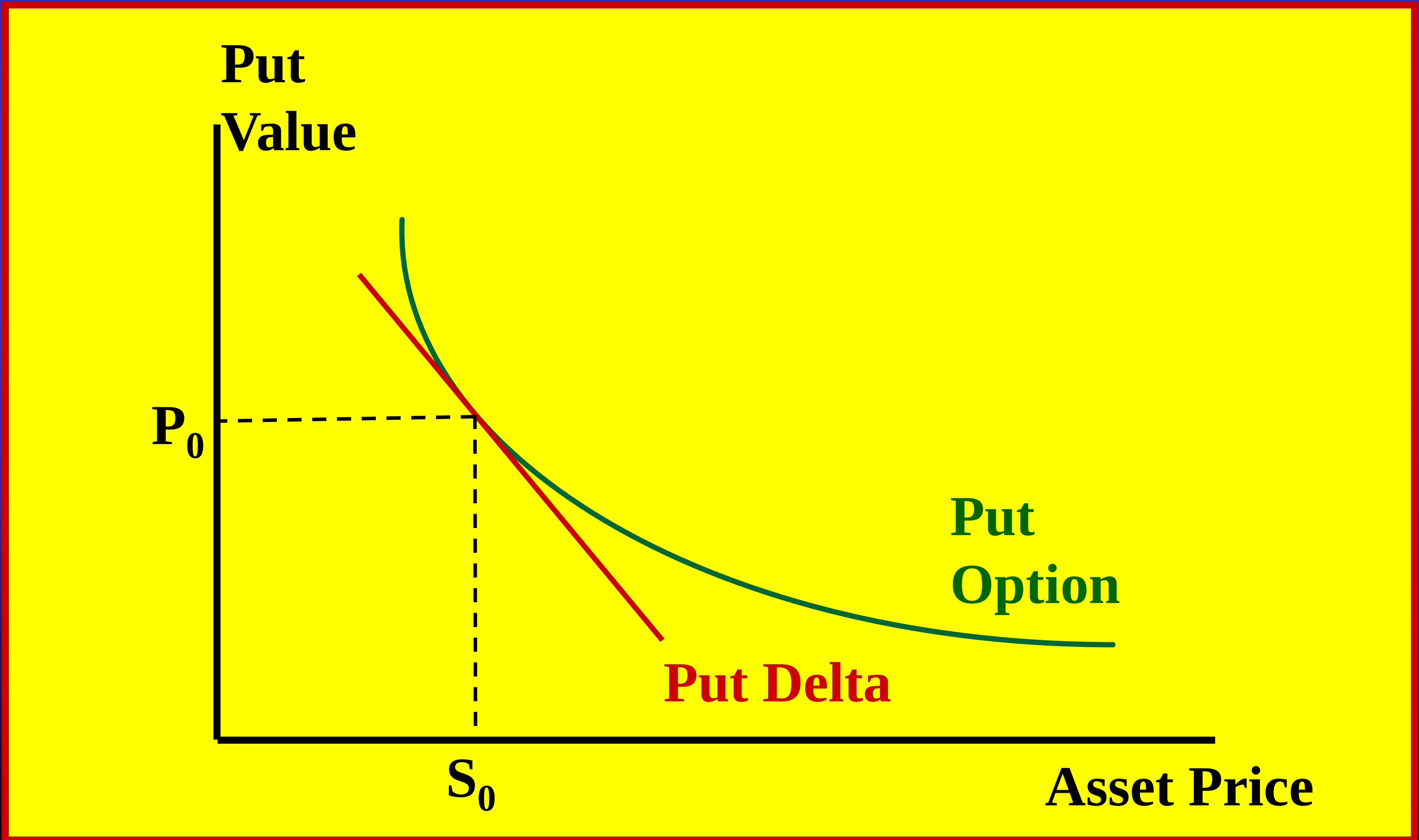
Call Option Sensitivity at a Given Market Level



Option Sensitivities

LOS 1 C b) Put Option Delta (Kolb pp. 424-428)

Put Option Sensitivity at a Given Market Level



Option Sensitivities

LOS 1 C c), d) Delta Formulas (Kolb pp. 423-428)

MATHEMATICAL FORMULAS:

$$\Delta_C = \frac{C_1 - C_0}{S_1 - S_0} \qquad \Delta_P = \frac{P_1 - P_0}{S_1 - S_0}$$

OR:

$$(C_1 - C_0) = \Delta_C \times (S_1 - S_0)$$

$$(P_1 - P_0) = \Delta_P \times (S_1 - S_0)$$

Option Sensitivities

LOS 1 C c), d) Delta Formulas (Kolb pp. 423-428)

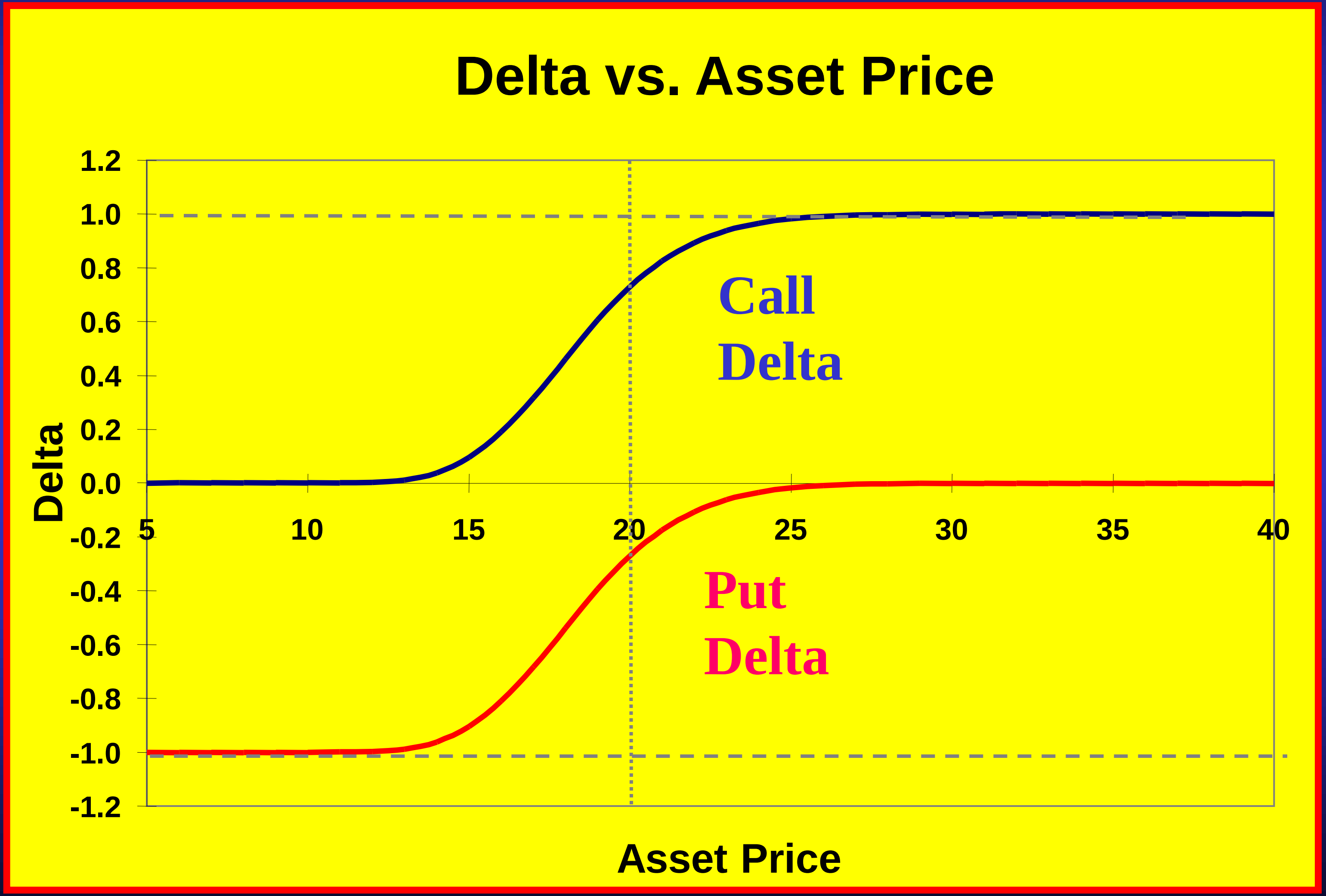
CALCULATING DELTA: SAMPLE PROBLEM

A put option falls from $4 in price to $3 in price when the stock rises from $72 to $75. What is the put option's delta?

$$\Delta_P = \frac{P_1 - P_0}{S_1 - S_0} = \frac{\$3 - \$4}{\$75 - \$72} = -0.33 = -33\%$$

Option Sensitivities

LOS 1 C f) The Behavior of Delta (Kolb pp. 429, 431)



Option Sensitivities

LOS 1 C f) The Behavior of Delta (Kolb pp. 429, 431)

Important Points

For Call Delta:

☞ Call delta rises from 0.0 (deep out of the money) to 1.0 (deep in the money) with a rising asset price

☞ Why? Higher asset price means the call is more likely to expire in the money, which means the call becomes more like the asset.

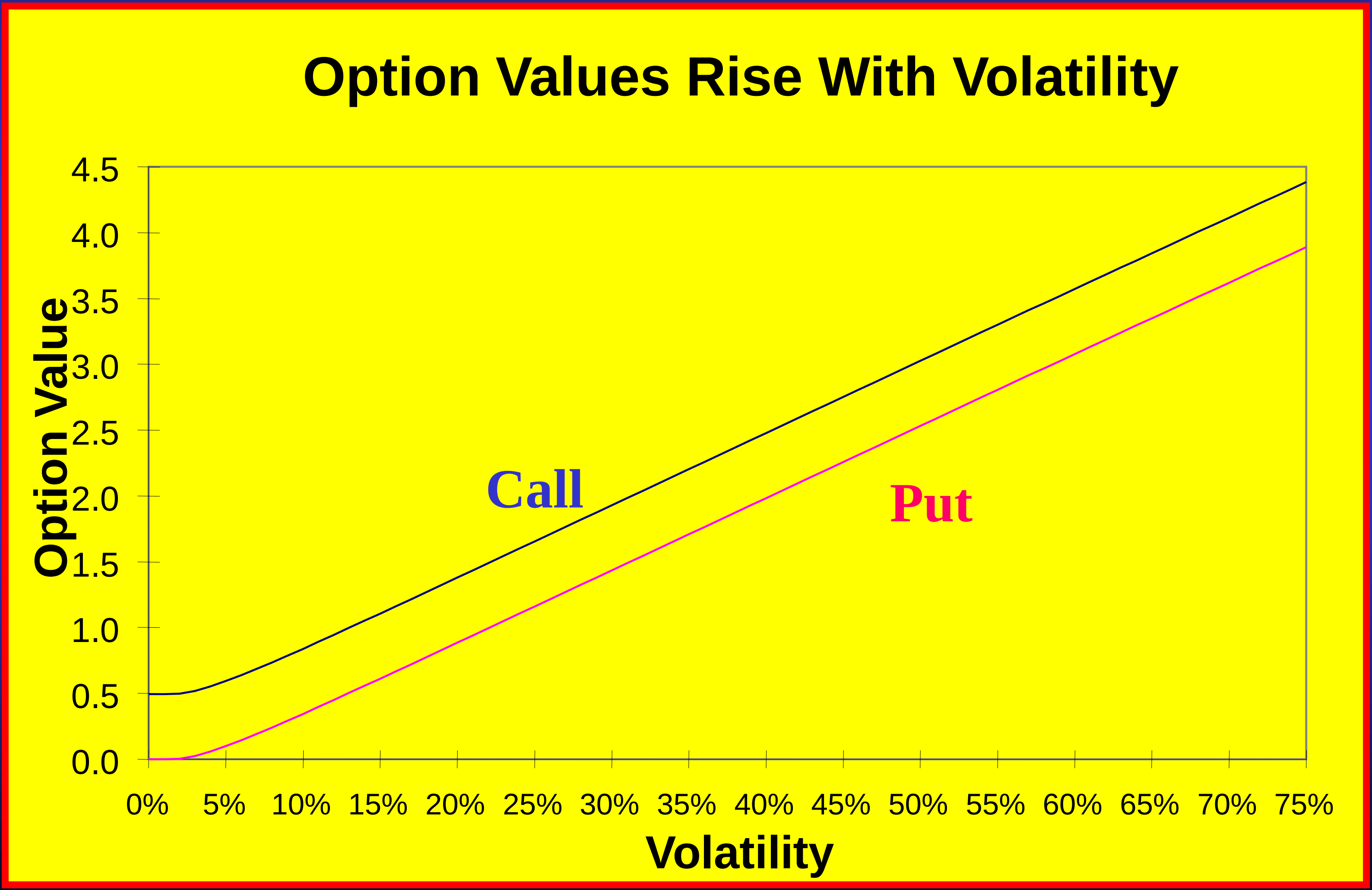
For Put Delta:

☞ Put delta rises from -1.0 (deep in the money) to 0.0 (deep out of the money) with a rising asset price

☞ Why? Higher asset price means the put is more likely to expire out of the money, which means it looks less like the asset.

Option Sensitivities

LOS 1 C g) Impact of Volatility (Kolb pp. 431-432)



Option Sensitivity

LOS 1 C g) Impact of Volatility (Kolb pp. 431-432)

<u>Why Does Option Value Rise with Increasing Volatility?</u>

For Call Options:

☞ If volatility is higher there is a greater chance the underlying asset will have a *higher* value at expiration (i.e., a good thing for calls)

☞ If volatility is higher there is a greater chance the underlying asset will have a *lower* value at expiration (can't hurt the call any further)

For Put Options:

☞ If volatility is higher there is a greater chance the underlying asset will have a *lower* value at expiration (i.e., a good thing for puts)

☞ If volatility is higher there is a greater chance the underlying asset will have a *higher* value at expiration (can't hurt the put any further)

Option Sensitivities

LOS 1 C g) Option Vega (Kolb pp. 431-432)

<u>Important Points About Vega</u>

☞ **Definition -** The sensitivity of an option's price to a change in the return volatility of the underlying asset

☞ **Other Names -** Vega is also known as kappa, lambda, or sigma

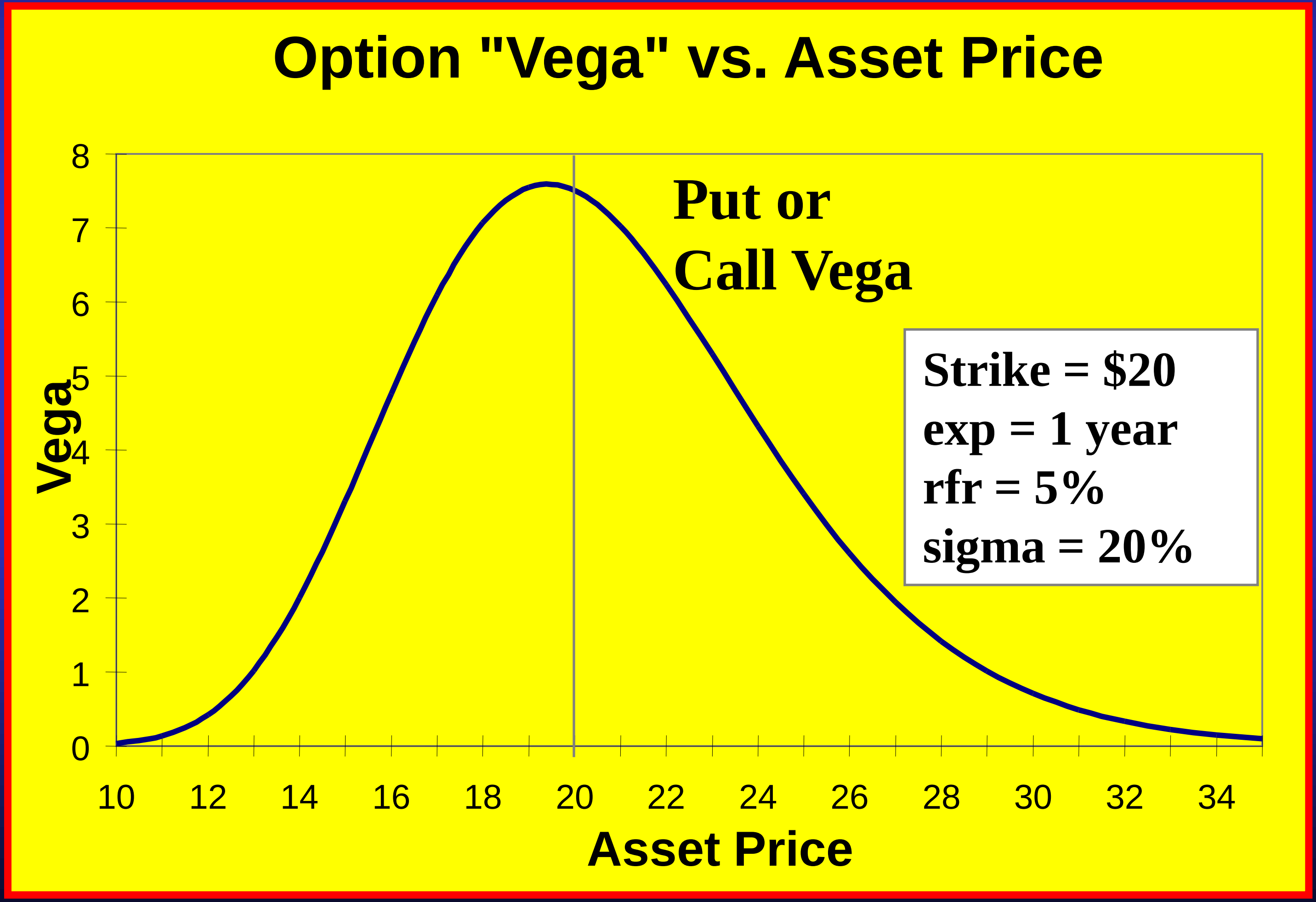
☞ **Standard Quote -** Vega is typically quoted in units of the option's price (i.e., in dollars) relative to a 1% change in asset volatility

☞ **Put Vega vs. Call Vega -** All other things equal, a put option will have the same vega as an equivalent call option

☞ **Infinitesimal Changes -** Like all other option derivatives, vega is only accurate for small changes in an option's volatility

Option Sensitivities

LOS 1 C h) Behavior of Vega (Kolb pp. 431-432, 436)



Option Sensitivities

LOS 1 C h) Behavior of Vega (Kolb pp. 431-432, 435-436)

<u>Important Points about Vega and Asset Price</u>

☞ **When does vega max?** - Vega is highest for near-the-money options

☞ **Can vega be negative?** - Vega is always positive

☞ **What happens away from the money?** - Vega is lower for out-of-the-money and in-the-money options than for at- or near-the-money options

☞ **Why the shape?** - Since the strike defines the point at which an option will have value or not, volatility makes the greatest difference at the strike.

Option Hedging

LOS 1 C e) Delta Neutral Hedging (Kolb pp. 428-430)

The Mechanics of Delta Neutral Hedging

☞ **Delta of the Underlying Asset** - The delta of the underlying asset is always 1.0

☞ **Delta of a Portfolio -** The delta of a portfolio is the weighted average of the deltas of the elements of the portfolio

☞ **Reducing Delta -** The delta of a portfolio can always be reduced by including something in the portfolio that has a negative delta

☞ **What has a Negative Delta?** - Short calls, short assets, and long puts have negative deltas.

Option Hedging

LOS 1 C e) Delta Neutral Hedging (Kolb pp. 428-430)

The Logic of Delta Neutral Hedging

☞ **The Theory** - A delta neutral portfolio does not change its value as the price of the asset rises or falls

☞ **The Limitations -** A delta neutral portfolio is only “neutral” for very small moves in the underlying asset price

☞ **Overcoming the Limitation** - If the asset price move is large, you must rebalance the portfolio (buy/sell options and or the asset)

☞ **Real World Considerations** - More frequent rebalancings back to neutrality provide greater protection but incur higher transaction costs

Option Hedging

LOS 1 C e) Delta Neutral Hedging (Kolb pp. 428-430)

An Example of Delta Neutral Hedging

Assume you own 1000 shares of stock. Call options exist with a delta of 0.8, and put options exist with a delta of -0.25. Without buying or selling any of the stock:

1. Form a delta-neutral hedge using only the calls. Answer: Combine the 1000 shares of stock with -1000 x (1/.8) = -1250 calls

2. Form a delta neutral portfolio using only the puts. Answer: Combine the 1000 shares of stock with -1000 x (1/-0.25) = 4000 puts.

Option Hedging

LOS 1 C i) Delta & Synthetic Calls (Kolb pp. 428, 383)

Relationship Between Delta-Neutral & Synthetic Calls

A Delta-Neutral Portfolio says, with continuous rebalancing:

Bond = -Call + Stock x Hedge Ratio

Synthetic Call says(see Binomial Pricing), with continuous rebalancing:

Call = -Bond + Stock x Hedge Ratio

It is just a re-arrangement of the same formula

Option Hedging

LOS 1 C j) Delta & Portfolio Insurance (Kolb pp. ???)

Using Delta to Replicate Portfolio Insurance

☞ **Setting a Floor and Horizon -** Pick a call with a strike price equal to a floor you want for your portfolio and an expiration equal to your protection horizon

☞**Mimicking Delta -** The call will have a delta (say 60%), and a portfolio of 60% stocks and 40% bonds will have comparable market exposure.

☞**Rebalancing -** As the market falls the delta declines. You can rebalance the stock/bond portfolio similarly and lower its exposure. It will never fall below the protective floor.

Option Sensitivities

LOS 1 C k)   Gamma (Kolb pp. 433-437)

Relationship of Gamma to Delta and Stock Price

$$\Gamma_C = \frac{\Delta_{C1} - \Delta_{C0}}{S_1 - S_0} = \Gamma_P = \frac{\Delta_{P1} - \Delta_{P0}}{S_1 - S_0}$$

WHERE:

$$\Delta_C = \frac{C_1 - C_0}{S_1 - S_0} \qquad \Delta_P = \frac{P_1 - P_0}{S_1 - S_0}$$

OPTIONS ON INDEXES, CURRENCIES, AND FUTURES

Options on Indexes and Futures

LOS 1 D a) Option Pricing (Kolb pp. 382-386, 396-402)

LOS 1 D a)

Calculate the value of a European call (put) option given either the Black-Scholes model or the Binomial model when no continuous dividend is paid by the underlying security

RESPONSE

THIS IS THE SAME QUESTION AS WAS ASKED AND ANSWERED IN LOS 1 B c). THERE IS NOTHING NEW TO ADD HERE

Options on Indexes and Futures

LOS 1 D b), c), e) Merton's Model (Kolb p. 487)

Merton's Adjustment to Black-Scholes for "Leakage"

If the asset underlying a European option is expected to lose value continuously over the life of the option, adjust the option pricing formula for the continuous "leakage" by substituting:

$$e^{-\delta(T-t)}S_t$$

which is the final reduced value of S_t, anywhere in the formula where you would have used S_t. Note that δ is the annualized continuous rate of leakage (e.g., the continuous dividend rate).

Answer to 1 D e): Note that Merton's model becomes the Black-Scholes model when $\delta = 0$.

Options on Indexes and Futures

LOS 1 D b), c) Merton's Model (Kolb p. 487)

<u>Merton's Adjustment to Black-Scholes Model: Formulas</u>

$$C_t^M = e^{-\delta(T-t)} S_t N\left(d_1^M\right) - Xe^{-r(T-t)} N\left(d_2^M\right)$$

where:

$$N(\cdot) = Cumulative\ Normal\ Distribution\ Function$$

$$d_1^M = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r - \delta + .5\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2^M = d_1^M - \sigma\sqrt{T-t}$$

Options on Indexes and Futures

LOS 1 D c), f) Binomial & Dividends (Kolb pp. 487-488)

Adjusting the Binomial Model for "Leakage"

The adjustment procedure requires using versions of the binomial formula not previously introduced in your required readings. They are presented below (see pages 384-385,394-395,405-406):

$\underline{NO\ LEAKAGES\ (\delta = 0)}$

$$U = e^{\sigma\sqrt{(T-t)}}$$

$$D = \frac{1}{U}$$

$$\pi_U = \frac{e^{r(T-t)} - D}{U - D}$$

$$\pi_D = 1 - \pi_U$$

$$C = \frac{\pi_U C_U + \pi_D C_D}{r}$$

$\underline{LEAKAGES\ (\delta > 0)}$

$$U = e^{\sigma\sqrt{(T-t)}}$$

$$D = \frac{1}{U}$$

$$\pi_U = \frac{e^{(r-\delta)(T-t)} - D}{U - D}$$

$$\pi_D = 1 - \pi_U$$

$$C = \frac{\pi_U C_U + \pi_D C_D}{r}$$

Options on Indexes and Futures

LOS 1 D c), f) Binomial & Dividends (Kolb pp. ????)

Convergence of Binomial Model to Merton Model

The Binomial Model adjusted for dividends will converge to the Merton model under the same conditions that the regular Binomial Model (not adjusted for dividends) converges to the Black-Scholes Model. That is, when you take the multi-Period binomial model to the limit of an infinite number of infinitely small time periods, you will get the Merton Model.

Note: This fact is not specifically mentioned or proven in the text.

Options on Indexes and Futures

LOS 1 D c) Binomial with Dividends (Kolb p. *406*)

Let:

U = 1 + percentage increase in the stock price

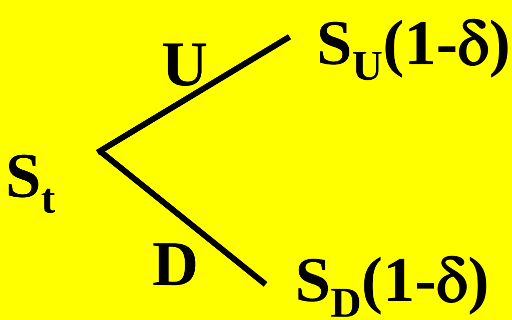
D = 1 - percentage decrease in the stock price

R = 1 + the riskless one period interest rate

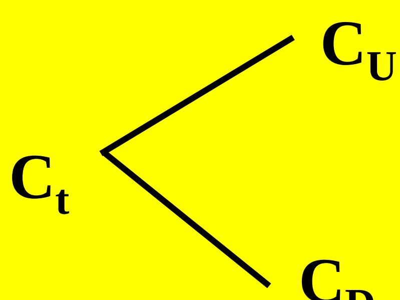
C_t = the current value of a call option

δ = the continuous leakage rate

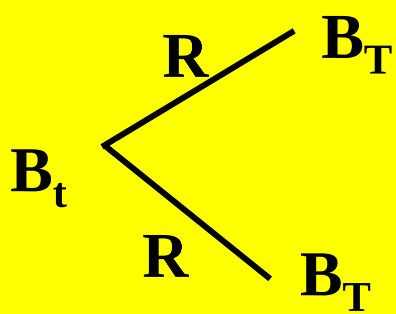
STOCK



CALL



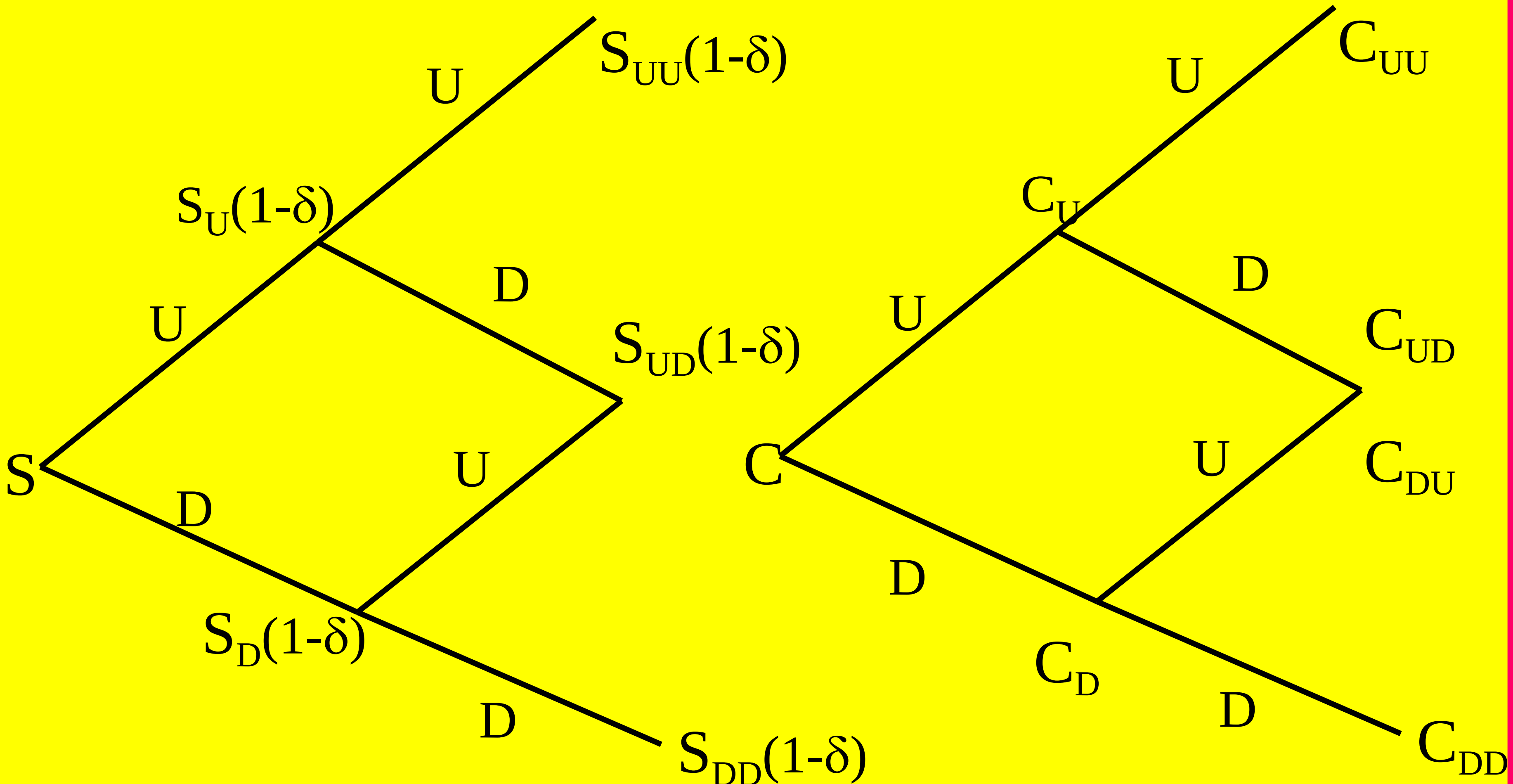
BOND



Options on Indexes and Futures

LOS 1 D c) Binomial with Dividends (Kolb p. *406*)

A Two-Period Binomial Pricing Model with Dividends



Options on Indexes and Futures

LOS 1 D g) Applying Merton's Model (Kolb pp. 488-493)

Applications of Merton's Model

If an asset decreases in value ("leaks") at a continuous rate:

Stock Index - With a large number of assets paying dividends at all different times, a stock index effectively pays a continuous dividend, which becomes the continuous dividend rate δ.

Futures - A futures contract is equivalent to the underlying asset less the ongoing cost of carrying the asset to expiration. The cost of carry is usually the same as the short term interest rate, so the continuous dividend rate equals the short-term rate (δ=r)

Currencies - Holding a foreign currency is like a stock: you own an asset that pays out a dividend (foreign interest rate),. The currency weakens ("leaks") at the continuous rate of the foreign interest rate

Options on Indexes and Futures

LOS 1 D b), c), g) Index Options (Kolb pp. 488-489)

Index Option Pricing: Example

You are looking at buying a 9 month European call option on the S&P 500 with a strike price of 1300. Current short-term interest rates are 7%, the index is currently at 1275, it has an annualized dividend rate of 2% and the annualized volatility of the index's returns is 18%.

What should you expect to pay for this European call option under a no-arbitrage pricing scenario?

Options on Indexes and Futures

LOS 1 D b), c), g) Index Options (Kolb pp. 488-489)

Index Option Pricing: Answer

First, identify from the problem the appropriate inputs to the Merton version of the Black-Scholes formula:

S_t = 1275   r = 7%   X = 1300   σ = 18%   (T-t) = 0.75   δ = 2%

Second, plug the values into the formula, starting with d^M_1 and d^M_2:

$$d_1^M = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r - \delta + .5\sigma^2\right)(T - t)}{\sigma\sqrt{T - t}}$$

$$= \frac{\ln\left(\frac{1275}{1300}\right) + \left(0.07 - 0.02 + 0.5 \times 0.18^2\right)(0.75)}{0.18 \times \sqrt{0.75}} = \frac{-0.01942 + 0.04965}{0.15588} = .1939$$

$$d_2^M = d_1^M - \sigma\sqrt{T - t} = .1939 - 0.18 \times \sqrt{0.75} = 0.03805$$

Options on Indexes and Futures

LOS 1 D b), c), g) Index Options (Kolb pp. 488-489)

Index Option Pricing: Answer

Finally, substitute the values for d^M_1 and d^M_2, along with other necessary inputs, into the main Merton equation:

$$
\begin{aligned}
C_t^M &= e^{-\delta(T-t)} S_t N\left(d_1^M\right) - Xe^{-r(T-t)} N\left(d_2^M\right) \\
&= e^{-0.02\times0.75} \times 1275 \times N(0.1939) - 1300 \times e^{-0.07\times0.75} \times N(0.03805) \\
&= 0.9851 \times 1275 \times 0.57689 - 1300 \times 0.94885 \times 0.51518 \\
&= 724.5753 - 635.4771 \\
&\approx 89.104
\end{aligned}
$$

Options on Indexes and Futures

LOS 1 D b), c), g) Futures Options (Kolb pp. 490-493)

Futures Option Pricing: Example

You are looking to buy a 1 year European futures call option on an S&P 500 futures contract. Current short-term interest rates are 5%, the S&P 500 index is currently at 1425, the annualized volatility of the index's returns is 15%, and you are considering a futures call option with a strike at 1400.

What should you expect to pay for this European call option under a no-arbitrage pricing scenario?

Options on Indexes and Futures

LOS 1 D b), c), g) Futures Options (Kolb pp. 490-493)

Futures Option Pricing: Answer

First, identify from the problem the appropriate inputs to the Black version of the Black-Scholes formula:

S_t = 1425   r = 5%   X = 1400   σ = 15%   (T-t) = 1.00

Second, plug values into the formula, starting with F_t, d^F_1 and d^F_2:

$$F_t = S_t e^{r(T-t)} = 1425 \times e^{0.05 \times 1.00} = 1498.0613$$

$$d_1^F = \frac{\ln\left(\frac{F_t}{X}\right) + (.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$= \frac{\ln\left(\frac{1498.0613}{1400}\right) + (0.5 \times 0.15^2)(1.00)}{0.15 \times \sqrt{1.00}} = \frac{0.0677 + 0.01125}{0.1500} = .52633$$

$$d_2^F = d_1^F - \sigma\sqrt{T-t} = .52633 - 0.15 \times \sqrt{1.00} = 0.37633$$

Options on Indexes and Futures

LOS 1 D b), c), g) Futures Options (Kolb pp. 490-493)

Index Option Pricing: Answer

Finally, substitute the values for d^F_1 and d^F_2, along with other necessary inputs, into the main Black equation:

$$
\begin{aligned}
C_t^F &= e^{-r(T-t)}\left[F_t N\left(d_1^F\right) - XN\left(d_2^F\right)\right] \\
&= e^{-0.05\times 1.00} \times \left[1498.0613 \times N\left(0.52633\right) - 1400 \times N\left(0.37633\right)\right] \\
&= 0.9512 \times \left[1498.0613 \times 0.70067 - 1400 \times 0.646664\right] \\
&= 0.9512 \times \left[1049.6466 - 905.3296\right] \\
&\approx 137.279
\end{aligned}
$$

Options on Indexes and Futures

LOS 1 D b), c), g) Currency Options (Kolb pp. 489-491)

COMMENT

The material in Kolb on Currency Options, which is also the only application in this chapter of binomial option pricing with dividends, is far more complicated than anything previously presented or covered in the Level II readings, such as multi-period binomial pricing and the log-normal distribution.

I doubt that the Examiners will test from this material. The more likely question would be on the Merton or Black models.

INTEREST RATE DERIVATIVE INSTRUMENTS

Interest Rate Derivatives

LOS 2 a) Basic Features (Fabozzi, pp. 564-565)

Basic Features of Interest Rate Futures Contracts

☞ **Definition -** An agreement to buy or sell an interest rate instrument (or cash equivalent) at a future date at a given price.

☞ **Settlement Date -** Future date at which parties must transact

☞ **Clearinghouse -** All futures contract clear through a clearinghouse

☞ **Margin Requirements** - Rate futures contracts have the same types of margin requirements as other futures contract - maintenance, variation, and initial margin.

☞ **Delivery Options -** Rate futures contracts offer the buyer the option to deliver one of a basket of interest rate instruments

Interest Rate Derivatives

LOS 2 b) Forwards vs. Futures (Fabozzi, pp. 565-566)

Contract Characteristic	Forwards	Futures
Contracts Traded on:	OTC	Exchange
Contract Design:	Flexible	Standardized
Performance Guaranteed By:	Counterparty	Clearinghouse
Parties Post Margin?	No	Yes
Government Regulated?	No	Yes
Marked to Market?	***No***	***Yes***
Need to Roll?	***No***	***Yes***
Liquidity?	***Thin***	***Good***

Interest Rate Derivatives

LOS 2 d) Counterparty Risk (Fabozzi, pp. 564-565)

CounterParty Risk in Forward and Futures Contracts

Definition - Counterparty risk is the risk that any of the parties to a forward/futures contract will fail to comply with the terms of the contract, causing financial harm to the other party.

Comments

(1) Marking to market helps alleviate counterparty risk

(2) Both parties face counterparty risk

(3) Futures contracts face less (but not zero) counterparty risk because the counterparty to every futures contract is the clearinghouse.

(4) Posting margin does not eliminate counterparty risk.

Interest Rate Derivatives

LOS 2 c) Treasury Bond Futures (Fabozzi, pp. 564-565)

Basic Features of Treasury Bond Futures Contracts

☞ **Definition -** An interest rate futures contract traded on the Chicago Board of Trade (CBT) that tracks a "Notional Bond"

☞ **Notional Bond -** A theoretical bond that has exactly 20 years to maturity, a $100,000 par value, and pays a 6% coupon

☞ **Price Quotes -** The bond future is quoted in 1/32's, with par being 100, meaning that a price of 97-16 actually means 97 16/32, or $97.50

☞ **Delivery Months** - Bond futures have delivery months on a March/June/September/December cycle

☞ **Deliverable Basket** - Bond futures can deliver any bond with 15 years or more to maturity or first call.

Interest Rate Derivatives

LOS 2 c) Treasury Bond Futures (Fabozzi, pp. 564-565)

Delivery Options of Treasury Bond Futures Contracts

☞ **Conversion Factor -** The CBT assigns a conversion factor to every deliverable bond to make it comparable to the notional bond.

☞ **Delivery Option -** The bond futures seller has the option to choose and deliver, and the buyer has the obligation to receive, any deliverable bond

☞ **Cheapest-to-Deliver (CTD) -** At any time in the life of the futures contract, including at settle, one bond in the deliverable basket will be cheaper to deliver by the seller to the buyer than all others due to pricing anomalies.

☞ **Invoice Price** - When delivered, the buyer pays an invoice price as determined by the following formula:

Invoice Price=(Contract Size x(Futures Settlement Price)x(Conversion Factor)
+ Accrued Interest on the Deliverable Bond

Interest Rate Derivatives

LOS 2 e) Basic Features (Fabozzi, pp. 564-565)

Basic Features of Interest Rate Options Contracts

☞ **Definition -** An option (put or call) that has as its underlying asset and interest rate instrument or an interest rate index.

☞ **Option Premium -** The cash amount paid in order to have the right to exercise the defined option

☞ **Strike Price -** The price or index level at which the option buyer can force a transaction to occur

☞ **Expiration Date** - The final date after which the rights to exercise the option no longer exists

☞ **Exercise Alternatives -** Options can be exercised only at settlement (European) or any time before settlement (American) or on selected dates before expiration (Bermuda or Atlantic)

Interest Rate Derivatives

LOS 2 f) Basic Features (Fabozzi, pp. 564-565)

Basic Features of Interest Rate Futures Options

☞ **Definition -** An option that gives its owner the right to be long (for a call) or short (for a put) a rate futures contract upon exercise.

☞ **Trading Mechanics -** Upon exercise, a futures contract is opened with the option writer and option buyer being assigned opposing positions. The future is priced at market, but the two counterparties are immediately marked to market relative to the exercise price.

☞ **Reasons for Popularity of Interest Rate Futures Options:**

(1) Don't have to deal with accrued interest
(2) Futures are plentiful and homogenous - no delivery squeezes
(3) Futures prices are widely available and distributed by the exchange

Interest Rate Derivatives

LOS 2 g) OTC Rate Options (Fabozzi, p. 578)

<u>Over-the-Counter (OTC) Interest Rate Options</u>

☞ **Definition -** An option specifically designed for an institutional investor that tracks one single government or agency bond

☞ **Trading Mechanics -** Just like a forward contract, two counterparties come together and agree on the terms of the option, including maturity, strike price, premium, underlying security, and expiration type

☞ **Reasons Institutional Investors use OTC Interest Rate Options:**

(1) The institutional investor currently has an exposure to that issue

(2) The institutional investor wants to hedge that specific issue

(3) The institutional investor does not want to deal basis risk or cross-hedging risk of hedging with a generic rate vehicle

Interest Rate Derivatives

LOS 2 h) Interest Rate Swaps (Fabozzi, p. 578-580)

LOS 2 h)

The LOS reviews the basics of plain vanilla interest rate swaps, including definitions, box diagrams, and cash flows.

THERE IS NOTHING IN THIS SECTION OF THE FABOZZI READING THAT IS NOT OTHERWISE COVERED IN THE APPROPRIATE SECTIONS ON INTEREST RATE SWAPS IN STUDY SESSION 17. PLEASE REFER TO THOSE SLIDES.

Interest Rate Derivatives

LOS 2 i) Caps and Floors (Fabozzi, pp. 584-585)

Basic Rate Options

Interest Rate Cap - A call option based on an interest rate index

Interest Rate Floor - A put option based on an interest rate index

Compound Rate Options

Interest Rate Collar -The purchase of an interest rate cap at a higher strike rate and the sale of an interest rate floor at a lower strike rate.

Interest Rate Corridor - The purchase of an interest rate cap at a given strike rate and the sale of another interest rate cap at a higher strike rate.

Interest Rate Derivatives

LOS 2 i) Descriptive Features (Fabozzi, pp. 584-585)

Defining Characteristics of Caps and Floors

Underlying Index - The interest rate index (e.g. LIBOR) that the rate option is valued against

Up-Front Premium - The price paid to the rate option seller by the rate option buyer

Notional Amount - The dollar amount that determines how much is paid when the rate option settles

Strike Rate - The strike price of the rate option, expressed in terms of the underlying index

Settlement Frequency - The number of times in the life of the rate option that the strike is valued against the underlying index.

Interest Rate Caps and Floors

LOS 4 a) Users of Rate Options (Fabozzi, pp. 584-586)

Who Uses Rate Options?

Interest Rate Caps - Investors that face floating rate liabilities might want to purchase a rate cap in order to cap a rising interest expense.

Interest Rate Floor - Investors that own floating rate assets might want to purchase a rate floor in order to establish a minimum rate of return that they will earn.

Interest Rate Collar - An investor considering the purchase of an interest rate cap can reduce its cost by selling an interest rate floor by giving up some of the gain that might be achieved if rates fall

Interest Rate Caps and Floors

LOS 4 b) Impact of Factors (Fabozzi, pp. 585-586)

Influence of Driving Factors on Rate Option Premium		
Impact of an increase in:	**INTEREST RATE CAP**	**INTEREST RATE FLOOR**
Rate Volatility	Increase	Increase
Strike Rate	Decrease	Increase
Term to Maturity	Increase	Increase
Payment Frequency	Increase	Increase

Interest Rate Derivatives

LOS 2 j) Cap/Floor Payoffs (Fabozzi, pp. 584-585)

Cash Flows in an Interest Rate Cap

$$\text{Settlement Payment} = \left(\text{Index Rate} - \text{Strike Rate}\right) \times \frac{\text{Days}}{360} \times \text{Notional Amount}$$

Cash Flows in an Interest Rate Floor

$$\text{Settlement Payment} = \left(\text{Strike Rate} - \text{Index Rate}\right) \times \frac{\text{Days}}{360} \times \text{Notional Amount}$$

Interest Rate Derivatives

LOS 2 j) Cap/Floor Payoffs (Fabozzi, pp. 584-585)

Cash Flows in an Interest Rate Cap: Example

An interest rate cap has a notional amount of $50,000,000, a quarterly settlement frequency, a strike rate of 5%, and an underlying index of LIBOR. If LIBOR is 7%, calculate the quarterly settlement payment.

$$\begin{matrix}\textit{Settlement}\\ \textit{Payment}\end{matrix} = \left(\begin{matrix}\textit{Index}\\ \textit{Rate}\end{matrix} - \begin{matrix}\textit{Strike}\\ \textit{Rate}\end{matrix}\right) \times \frac{\textit{Days}}{360} \times \begin{matrix}\textit{Notional}\\ \textit{Amount}\end{matrix}$$

$$= (.07 - .05) \times \frac{90}{360} \times \$50{,}000{,}000$$

$$= \$250{,}000$$

Interest Rate Derivatives

LOS 2 k) Rate vs. Bond Options (Fabozzi, pp. 585-586)

Distinction between Rate Options and Bond Options

(1) Options on interest rates move directly with interest rates.

(2) Options on fixed income securities move directly with the price of the fixed income security.

(3) Prices of fixed income securities are negatively (and non-linearly) related to interest rates

CONCLUSION

	Rates Rise	Rates Fall
Rate Call (Cap)	Value Rises	Value Falls
Rate Put (Floor)	Value Falls	Value Rises
Instrument Call	Value Falls	Value Rise
Instrument Put	Value Rises	Value Falls

Interest Rate Derivatives

LOS 2 l) Cap/Floor Interpretations (Fabozzi, p. 585)

Interpretation of Cap and Floor Positions

- ☞ A Cap or a Floor can be interpreted as a package of separate interest rate options.
- ☞ A Cap is a multi-period sequence of **Caplets**. A Caplet is an interest rate call option on a reference rate (like LIBOR) for a single period.
- ☞ A Floor is a multi-period sequence of **Floorlets**. A Floorlet is an interest rate put option on a reference rate (like LIBOR) for a single period.

OPTIONS MODELS AND OPTION PRICES

Option Models vs. Option Prices

LOS 3 a) Why Arbitrage Fails (Figlewski, p. 12)

Main Arbitrage Assumption

☞ Model assumes option arbitrage is *riskless* and *costless.*

Arbitrage isn't riskless

☞ Can't rebalance when markets are closed

☞ Can't rebalance when markets "gap"

Arbitrage isn't costless

☞ Any Rebalancing has commission and market impact costs

☞ Fixed transactions costs affect optimal rebalance frequency

Conclusion

☞ Arbitrage traders don't "pounce" on all option mispricings

Option Models vs. Option Prices

LOS 3 b) Non-Black-Scholes Factors (Figlewski, p. 12)

<u>Option Influences Not in Black-Scholes</u>

- ☞ **Transactions Costs**
- ☞ **Taxation**
- ☞ **Margin Treatment of Different Securities**
- ☞ **Delivery Features of Options Contracts**
- ☞ **Interaction Between Options and Related Futures**
- ☞ **Constraints on Short Sales**
- ☞ **Other Supply/Demand Factors Influencing Investors**
  - ⌧Outlook for Market Direction
  - ⌧Hedging Needs of Investors
  - ⌧Desire for Income

Option Models vs. Option Prices

LOS 3 c) Black-Scholes' Flaws (Figlewski, p. 12)

Why Black-Scholes Might Be Wrong

☞ **Fat-Tailed Stock Return Distributions**

- Causes "gaps" that interfere with hedging
- Leads to undervaluation of short-maturity options

☞ **Non-Random Price Movements**

- Serial correlation has been observed
- Evidence found in both short-run and long-run

☞ **Non-Stationary Input Parameters**

- Volatility and interest rates may move over time
- Volatility may be correlated with market direction
- Longer-term options would then be mispriced

Option Models vs. Option Prices

LOS 3 d) Pricing vs. Valuation (Figlewski, p. 14)

Is Black-Scholes Right and the Market Wrong?

- ☞ Market-based option prices often differ from model values.
- ☞ Traders often find it difficult to conduct arbitrages
- ☞ Some academics claim this as proof B-S is "wrong"

The Real Test of Black-Scholes Should Be:

- ☞ If you could have transacted at theoretical option values
- ☞ If you could have performed the arb trade costlessly
- ☞ If such a trade returned the risk-free rate, then

Conclusion:

- ☞ The model is correct - it predicts what it is designed to predict
- ☞ Traders should use it for valuation, not for exact pricing