OPTION OPTIMIZATION PROGRAM

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TSA ANALYTICAL MODELS

Option Optimization

Purpose

- The *Option Optimization Program* uses a Linear Programming algorithm to find an optimal mix of option positions. This allows us to structure an option portfolio to meet specific risk/return profiles.

- Return potential is enhanced by specifying an objective function that identifies the exact source of the desired return.

- Risk is controlled by the imposition of a variety of constraints, as needed. Constraints can be loosened or tightened to meet a specific risk/return profile.

- Portfolio gains and losses can be decomposed into precise measures that identify which characteristics of the strategy are contributing to the overall return.

- The Option Optimization Program can also be used with existing client strategies, such as volatility trading, to find that position which implements the client's pre-existing strategy in the most profitable and risk-controlled manner.

Option Optimization Program

Investment Strategy

- The availability of closed-form pricing equations for many of the most popular option contracts provides accurate assessments of short-term exposures to an option position .
- Many of the higher order comparative statics of the option pricing equations (i.e., the "greeks") provide valuable information, but are underutilized in the derivatives community.
- For an underlying instrument that has a wide array of actively traded strikes and expirations (stock indices, Eurodollars, key currencies), the derivatives of each option can be linearly combined to form any number of constraints and/or objective functions.
- Mathematical programming techniques such as Linear Programming are used to optimize across any feasible combination of these constraints and objective functions.
- Real-time tracking of the constraints and objectives can be used within a Taylor series approximation framework to identify when the option portfolio has drifted "too far" away from its initial optimization, allowing for both precise factor attribution and for signalling the portfolio re-optimization.

Option Optimization Program

Investment Strategy

Choice of Decision Variables

In every optimization a decision has to be made as to what variables will be optimized. These would include such variables as:

- The actual number of contracts to be bought or sold on a given underlying instrument. Alternatively, the percent of a given notional amount that is to be invested in each option.
- The number of contracts of the underlying asset, if feasible. This can be valuable in expanding the universe of instruments that can be used to meet each constraint, thus allowing for a better optimum.
- Holdings of other closely related options and their underlying instruments. To the extent that two markets are closely correlated, their option sensitivities should remain approximately linearly additive.

TSA ANALYTICAL MODELS

Option Optimization

Choice of Objective Functions

The fundamental purpose of the Option Optimization Program is to allow the investor choice over the source of return while also controlling risk. Here are a few examples of how returns can be structured:

- **Theta Maximization** - Portfolios can be constructed to maximize time decay. Contrary to intuition, this may not involve holding options with large time decay, particularly when risk considerations are taken into account. As variations on this theme, the portfolio can also be constructed in such a way that the vulnerability of portfolio time decay to movements in other market factors is minimized.

- **Optimal Risk Shifting** - An optimization can be constructed to shift risk from an event considered likely to one considered unlikely. For example, if a client believes the Fed will not alter short-term rates, exposure to swings in volatility (vega) or the underlying market (delta, gamma) can be reduced for a tradeoff of increasing exposure to interest rate changes, which are not deemed highly probable.

- **Vega Optimization** - Given a view on the direction of volatility, a portfolio can be constructed so as to maximize desired exposure to either an expected rise or an expected fall in volatility.

Additionally, each objective function can be designed to minimize transaction costs, or to recommend the best manner in which to roll the position forward beyond expiration dates.

Option Optimization

Choice of Risk Constraints

The effectiveness of any optimization is in the constraints. Constraints are needed not only to control risk, but also to provide for portfolio scale and to implement client-specific restrictions. Examples:

- **Scale Constraints** - Constraints can be added to limit the total number of contracts engaged to within tolerable limits, to generate an original cash inflow or outflow, to see that the number of longs and shorts remains in balance, or to see that no one option position exceeds a given limit.

- **Strategy Constraints** - An existing strategy can be easily accomodated. For example, an existing calendar condor position can be re-optimized to limit exposure to current markets.

- **Neutrality Constraints** - The standard neutrality conditions can be placed on the portfolio; such as, delta-, vega-, and rho-neutrality. Any neutrality constraint can be made one-sided to allow exposure to dowside (upside) moves in the market, interest rates, or volatility. Likewise, theta can be made positive or negative.

- **Stability Constraints** - The greatest flexibility comes from the judicious use of stability constraints. These constraints help prevent drift in the neutrality constraints, and provide the option position with a broader "flat-spot" in its exposure to underlying factors. For example, desired theta or vega levels can be protected from drift caused by market moves, volatility changes, or the passage of time.

Option Optimization

Sample Formulation

The following LP formulation seeks to maximize the VEGA sensitivity of an option portfolio while simultaneously immunizing the portfolio against market moves and vega's drift over time. Theta is also forced to be positive.

MAX **VEGA** $= \sum \mathbf{w}_1 \nu_1 + \mathbf{w}_2 \nu_2 + \ldots + \mathbf{w}_n \nu_n$

SUBJECT TO:

$\mathbf{w}_1 \delta_1 + \mathbf{w}_2 \delta_2 + \ldots + \mathbf{w}_n \delta_n = 0 \Rightarrow$ **delta set to zero**

$\mathbf{w}_1 \gamma_1 + \mathbf{w}_2 \gamma_2 + \ldots + \mathbf{w}_n \gamma_n = 0 \Rightarrow$ **gamma set to zero**

$\mathbf{w}_1 \theta_1 + \mathbf{w}_2 \theta_2 + \ldots + \mathbf{w}_n \theta_n \geq 0 \Rightarrow$ **theta forced to be positive**

$\mathbf{w}_1 \nu_{\tau_1} + \mathbf{w}_2 \nu_{\tau_2} + \ldots + \mathbf{w}_n \nu_{\tau_n} = 0 \Rightarrow$ **vega immunized against drift over time**

$\mathbf{w}_1^+ + \mathbf{w}_2^+ + \ldots \mathbf{w}_n^+ \leq \text{MAXLONG} \Rightarrow$ **constrain total long option position**

$\mathbf{w}_1^- + \mathbf{w}_2^- + \ldots \mathbf{w}_n^- \leq \text{MAXSHORT} \Rightarrow$ **constrain total short option position**

$\mathbf{w}_1^+ \geq 0 \quad \mathbf{w}_1^- \geq 0 \quad \cdots \mathbf{w}_n^+ \geq 0 \Rightarrow$ **decision variables must be nonnegative**

Option Optimization

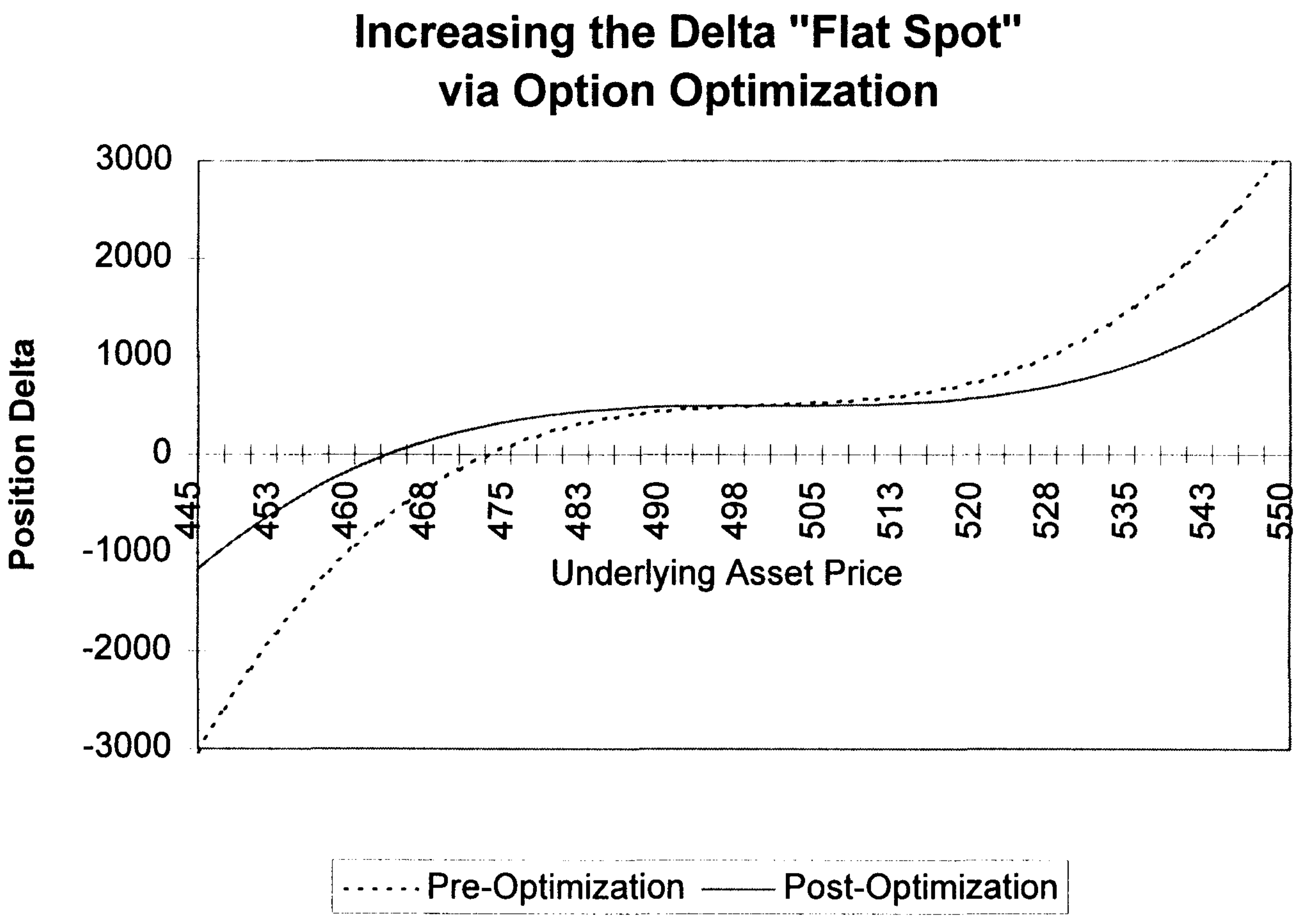

Diagram is for illustration only and does not represent an actual trade performed by TSA.

Optimization Result: Option positions are found with market senstivities that are "flatter" than a trader could discover just by intuition