Option Formulas

Valuations, First Derivatives
And Cross Derivatives

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Black-Scholes (1973) Option Model

Definition of Variables Used in the Black-Scholes (1973) Option Formula	
C =	Theoretical Call Option Price
P =	Theoretical Put Option Price
S =	Underlying Asset Spot Price
X =	Exercise (Strike) Price
σ =	Volatility of the Underlying Asset's Returns, Annualized
τ =	Time to Expiration, as a fraction of a year
r_D =	Risk-Free Domestic Interest Rate, Annualized
N(x) =	Cumulative Normal Probability Distribution evaluated at x
N´(x) =	Normal Probability Density Function evaluated at x

Intermediate Formulas (d_1 and d_2) for the The Black-Scholes (1973) Option Pricing Model	
$d_1 = \frac{\ln\left(\frac{S}{X}\right) + r_D\tau + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}$	$d_2 = \frac{\ln\left(\frac{S}{X}\right) + r_D\tau - \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}$
$N'(d_1) = N'(d_2)\left(\frac{X}{S}\right)e^{-r_D\tau}$	$N'(d_2) = N'(d_1)\left(\frac{S}{X}\right)e^{r_D\tau}$
$N''(d_i) = -d_i N'(d_i) \quad i = 1, 2$	

Option Pricing Formula for the Black-Scholes (1973) Option Pricing Model **Continuously priced cash-settled security, no dividend**
$C = SN(d_1) - e^{-r_D\tau} X N(d_2)$
$P = S\{N(d_1) - 1\} - e^{-r_D\tau} X \{N(d_2) - 1\}$

Black-Scholes (1973) Option Pricing Model Journal Citation
Black, Fischer and Scholes, Myron. (May-June 1973). “The Pricing of Options and Corporate Liabilities”. *Journal of Political Economy* **81** (3): 637-654.

Black-Scholes (1973) Model Useful Intermediate Derivatives	
$\frac{\partial d_1}{\partial S}=\frac{1}{S\sigma\sqrt{\tau}}$	$\frac{\partial d_2}{\partial S}=\frac{1}{S\sigma\sqrt{\tau}}$
$\frac{\partial d_1}{\partial X}=-\frac{1}{X\sigma\sqrt{\tau}}$	$\frac{\partial d_2}{\partial X}=-\frac{1}{X\sigma\sqrt{\tau}}$
$\frac{\partial d_1}{\partial r_D}=\frac{\sqrt{\tau}}{\sigma}$	$\frac{\partial d_2}{\partial r_D}=\frac{\sqrt{\tau}}{\sigma}$
$\frac{\partial d_1}{\partial \sigma}=-\frac{d_2}{\sigma}$	$\frac{\partial d_2}{\partial \sigma}=-\frac{d_1}{\sigma}$
$\frac{\partial d_1}{\partial \tau}=\frac{r_D}{\sigma\sqrt{\tau}}-\frac{d_2}{2\tau}$	$\frac{\partial d_2}{\partial \tau}=\frac{r_D}{\sigma\sqrt{\tau}}-\frac{d_1}{2\tau}$

OPTION FORMULAS

First Derivatives of the Black-Scholes (1973) Option Pricing Model		
Derivative with Respect to:	**Call Options**	**Put Options**
The Underlying Asset Price (S)	$\frac{\partial C}{\partial S} = N(d_1)$	$\frac{\partial P}{\partial S} = N(d_1) - 1$
The Option's Strike Price (X)	$\frac{\partial C}{\partial X} = -e^{-r_D \tau} N(d_2)$	$\frac{\partial P}{\partial X} = -e^{-r_D \tau} \{N(d_2) - 1\}$
The Annualized Domestic Risk Free Rate Of Return (r_D)	$\frac{\partial C}{\partial r_D} = -\tau X \frac{\partial C}{\partial X}$	$\frac{\partial P}{\partial r_D} = -\tau X \frac{\partial P}{\partial X}$
The Annualized Instantaneous Return Volatility of the Asset (σ)	$\frac{\partial C}{\partial \sigma} = S\sqrt{\tau} N'(d_1)$	$\frac{\partial P}{\partial \sigma} = S\sqrt{\tau} N'(d_1)$
The Option's Time to Expiry (τ)	$\frac{\partial C}{\partial \tau} = \frac{\sigma}{2\tau} \frac{\partial C}{\partial \sigma} + \frac{r_D}{\tau} \frac{\partial C}{\partial r_D}$	$\frac{\partial P}{\partial \tau} = \frac{\sigma}{2\tau} \frac{\partial P}{\partial \sigma} + \frac{r_D}{\tau} \frac{\partial P}{\partial r_D}$

Second Own Derivatives of the Black-Scholes (1973) Option Pricing Model		
Derivative with Respect to:	**Call Options**	**Put Options**
The Underlying Asset Price (S)	$\frac{\partial^2 C}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{\tau}}$	$\frac{\partial^2 P}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{\tau}}$
The Option's Strike Price (X)	$\frac{\partial^2 C}{\partial X^2} = \left(\frac{-S}{X}\right) \frac{\partial^2 C}{\partial S \partial X}$	$\frac{\partial^2 P}{\partial X^2} = \left(\frac{-S}{X}\right) \frac{\partial^2 P}{\partial S \partial X}$
The Annualized Domestic Risk Free Rate Of Return (r_D)	$\frac{\partial^2 C}{\partial {r_D}^2} = -\tau X \frac{\partial^2 C}{\partial X \partial r_D}$	$\frac{\partial^2 P}{\partial {r_D}^2} = -\tau X \frac{\partial^2 P}{\partial X \partial r_D}$
The Annualized Instantaneous Return Volatility of the Asset (σ)	$\frac{\partial^2 C}{\partial \sigma^2} = \frac{\partial C}{\partial \sigma} \frac{d_1 d_2}{\sigma}$	$\frac{\partial^2 P}{\partial \sigma^2} = \frac{\partial P}{\partial \sigma} \frac{d_1 d_2}{\sigma}$
The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial \tau^2} = \frac{\sigma}{2\tau^2} \left[\tau \frac{\partial^2 C}{\partial \sigma \partial \tau} - \frac{\partial C}{\partial \sigma}\right] - r_D X \frac{\partial^2 C}{\partial X \partial \tau}$	$\frac{\partial^2 P}{\partial \tau^2} = \frac{\sigma}{2\tau^2} \left[\tau \frac{\partial^2 P}{\partial \sigma \partial \tau} - \frac{\partial P}{\partial \sigma}\right] - r_D X \frac{\partial^2 P}{\partial X \partial \tau}$

Second Cross Derivatives of the Black-Scholes (1973) Option Pricing Model		
Derivative with Respect to:	**Call Options**	**Put Options**
Asset Price (S) and Strike Price (X)	$\frac{\partial^2 C}{\partial S \partial X} = -\frac{N'(d_1)}{X\sigma\sqrt{\tau}}$	$\frac{\partial^2 P}{\partial S \partial X} = -\frac{N'(d_1)}{X\sigma\sqrt{\tau}}$
Asset Price (S) and Domestic Risk Free Rate (r_D)	$\frac{\partial^2 C}{\partial S \partial r_D} = \frac{N'(d_1)\sqrt{\tau}}{\sigma}$	$\frac{\partial^2 P}{\partial S \partial r_D} = \frac{N'(d_1)\sqrt{\tau}}{\sigma}$
Asset Price (S) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial S \partial \sigma} = -\frac{N'(d_1)d_2}{\sigma}$	$\frac{\partial^2 P}{\partial S \partial \sigma} = -\frac{N'(d_1)d_2}{\sigma}$
Asset Price (S) and Time to Expiry (τ)	$\frac{\partial^2 C}{\partial S \partial \tau} = N'(d_1)\left[\frac{r_D}{\sigma\sqrt{\tau}} - \frac{d_2}{2\tau}\right]$	$\frac{\partial^2 P}{\partial S \partial \tau} = N'(d_1)\left[\frac{r_D}{\sigma\sqrt{\tau}} - \frac{d_2}{2\tau}\right]$
Strike Price (X) and Domestic Risk Free Rate (r_D)	$\frac{\partial^2 C}{\partial X \partial r_D} = -\tau\left[X\frac{\partial^2 C}{\partial X^2} + \frac{\partial C}{\partial X}\right]$	$\frac{\partial^2 P}{\partial X \partial r_D} = -\tau\left[X\frac{\partial^2 P}{\partial X^2} + \frac{\partial P}{\partial X}\right]$
Strike Price (X) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial X \partial \sigma} = -Sd_1\sqrt{\tau}\frac{\partial^2 C}{\partial S \partial X}$	$\frac{\partial^2 P}{\partial X \partial \sigma} = -Sd_1\sqrt{\tau}\frac{\partial^2 P}{\partial S \partial X}$
Strike Price (X) and the Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial X \partial \tau} = \frac{\sigma}{2\tau}\frac{\partial^2 C}{\partial X \partial \sigma} + \frac{r_D}{\tau}\frac{\partial^2 C}{\partial X \partial r_D}$	$\frac{\partial^2 P}{\partial X \partial \tau} = \frac{\sigma}{2\tau}\frac{\partial^2 P}{\partial X \partial \sigma} + \frac{r_D}{\tau}\frac{\partial^2 P}{\partial X \partial r_D}$
Domestic Risk Free Rate (r_D) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial r_D \partial \sigma} = -\tau X\frac{\partial^2 C}{\partial X \partial \sigma}$	$\frac{\partial^2 P}{\partial r_D \partial \sigma} = -\tau X\frac{\partial^2 P}{\partial X \partial \sigma}$
Domestic Risk Free Rate (r_D) and The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial r_D \partial \tau} = -X\left(\tau\frac{\partial^2 C}{\partial X \partial \tau} + \frac{\partial C}{\partial X}\right)$	$\frac{\partial^2 P}{\partial r_D \partial \tau} = -X\left(\tau\frac{\partial^2 P}{\partial X \partial \tau} + \frac{\partial P}{\partial X}\right)$
Annualized Return Volatility (σ) and The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial \sigma \partial \tau} = \frac{1}{2\tau}\left[\frac{\partial C}{\partial \sigma} + \frac{\partial^2 C}{\partial \sigma^2}\right] + \frac{\partial^2 C}{\partial r_D \partial \sigma}$	$\frac{\partial^2 P}{\partial \sigma \partial \tau} = \frac{1}{2\tau}\left[\frac{\partial P}{\partial \sigma} + \frac{\partial^2 P}{\partial \sigma^2}\right] + \frac{\partial^2 P}{\partial r_D \partial \sigma}$

Black (1976)
Option Model

Definition of Variables Used in the Black (1976) Option Formula	
C =	Theoretical Call Option Price
P =	Theoretical Put Option Price
F =	Underlying Asset Forward/Future Price
X =	Exercise (Strike) Price
σ =	Volatility of the Underlying Asset's Returns, Annualized
τ =	Time to Expiration, as a fraction of a year
r_{D} =	Risk-Free Domestic Interest Rate, Annualized
N(x) =	Cumulative Normal Probability Distribution evaluated at x
N´(x) =	Normal Probability Density Function evaluated at x

Intermediate Formulas (d_1 and d_2) for the Black (1976) Option Pricing Model	
$d_1 = \frac{\ln\left(\frac{F}{X}\right) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}$	$d_2 = \frac{\ln\left(\frac{F}{X}\right) - \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}$
$N'(d_1) = N'(d_2)\left(\frac{X}{F}\right)$	$N'(d_2) = N'(d_1)\left(\frac{F}{X}\right)$
$N''(d_i) = -d_i N'(d_i) \quad i = 1, 2$	

Option Pricing Formula for the Black (1976) Option Pricing Model Continuously priced forward-settled security, no dividend
$C = e^{-r_D\tau}\left(F N(d_1) - X N(d_2)\right)$
$P = e^{-r_D\tau}\left(F\{N(d_1) - 1\} - X\{N(d_2) - 1\}\right)$

Black (1976) Option Pricing Model **Journal Citation**
Black, Fischer. (March 1976). "The Pricing of Commodity Contracts", *Journal of Financial Economics* **3**: 167-179.

Useful Intermediate Derivatives for the Black (1976) Option Pricing Model	
$\frac{\partial d_1}{\partial F} = \frac{1}{F\sigma\sqrt{\tau}}$	$\frac{\partial d_2}{\partial F} = \frac{1}{F\sigma\sqrt{\tau}}$
$\frac{\partial d_1}{\partial X} = -\frac{1}{X\sigma\sqrt{\tau}}$	$\frac{\partial d_2}{\partial X} = -\frac{1}{X\sigma\sqrt{\tau}}$
$\frac{\partial d_1}{\partial \sigma} = -\frac{d_2}{\sigma}$	$\frac{\partial d_2}{\partial \sigma} = -\frac{d_1}{\sigma}$
$\frac{\partial d_1}{\partial \tau} = -\frac{d_2}{2\tau}$	$\frac{\partial d_2}{\partial \tau} = -\frac{d_1}{2\tau}$

OPTION FORMULAS

First Derivatives of the Black (1976) Option Pricing Model		
Derivative with Respect to:	**Call Options**	**Put Options**
The Underlying Futures Contract Price (F)	$\frac{\partial C}{\partial F} = e^{-r_D \tau} N(d_1)$	$\frac{\partial P}{\partial F} = e^{-r_D \tau} \{N(d_1) - 1\}$
The Option's Strike Price (X)	$\frac{\partial C}{\partial X} = -e^{-r_D \tau} N(d_2)$	$\frac{\partial P}{\partial X} = -e^{-r_D \tau} \{N(d_2) - 1\}$
The Annualized Domestic Risk Free Rate Of Return (r_D)	$\frac{\partial C}{\partial r_D} = -\tau C$	$\frac{\partial P}{\partial r_D} = -\tau P$
The Annualized Instantaneous Return Volatility of the Asset (σ)	$\frac{\partial C}{\partial \sigma} = F e^{-r_D \tau} \sqrt{\tau} N'(d_1)$	$\frac{\partial P}{\partial \sigma} = F e^{-r_D \tau} \sqrt{\tau} N'(d_1)$
The Option's Time to Expiry (τ)	$\frac{\partial C}{\partial \tau} = \frac{\sigma}{2\tau} \frac{\partial C}{\partial \sigma} + \frac{r_D}{\tau} \frac{\partial C}{\partial r_D}$	$\frac{\partial P}{\partial \tau} = \frac{\sigma}{2\tau} \frac{\partial P}{\partial \sigma} + \frac{r_D}{\tau} \frac{\partial P}{\partial r_D}$

Second Own Derivatives of the Black (1976) Option Pricing Model		
Derivative with Respect to:	**Call Options**	**Put Options**
The Underlying Futures Contract Price (F)	$\frac{\partial^2 C}{\partial F^2} = \frac{e^{-r_D \tau} N'(d_1)}{F \sigma \sqrt{\tau}}$	$\frac{\partial^2 P}{\partial F^2} = \frac{e^{-r_D \tau} N'(d_1)}{F \sigma \sqrt{\tau}}$
The Option's Strike Price (X)	$\frac{\partial^2 C}{\partial X^2} = \frac{e^{-r_D \tau} N'(d_2)}{X \sigma \sqrt{\tau}}$	$\frac{\partial^2 P}{\partial X^2} = \frac{e^{-r_D \tau} N'(d_2)}{X \sigma \sqrt{\tau}}$
The Annualized Domestic Risk Free Rate Of Return (r_D)	$\frac{\partial^2 C}{\partial {r_D}^2} = -\tau \frac{\partial C}{\partial r_D}$	$\frac{\partial^2 P}{\partial {r_D}^2} = -\tau \frac{\partial P}{\partial r_D}$
The Annualized Instantaneous Return Volatility of the Asset (σ)	$\frac{\partial^2 C}{\partial \sigma^2} = \frac{\partial C}{\partial \sigma} \frac{d_1 d_2}{\sigma}$	$\frac{\partial^2 P}{\partial \sigma^2} = \frac{\partial P}{\partial \sigma} \frac{d_1 d_2}{\sigma}$
The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial \tau^2} = \frac{\sigma}{2\tau^2} \left[\tau \frac{\partial^2 C}{\partial \sigma \partial \tau} - \frac{\partial C}{\partial \sigma} \right] - r_D \frac{\partial C}{\partial \tau}$	$\frac{\partial^2 P}{\partial \tau^2} = \frac{\sigma}{2\tau^2} \left[\tau \frac{\partial^2 P}{\partial \sigma \partial \tau} - \frac{\partial P}{\partial \sigma} \right] - r_D \frac{\partial P}{\partial \tau}$

Second Cross Derivatives of the Black (1976) Option Pricing Model

Derivative with Respect to:	Call Options	Put Options
Future Price (F) and Strike Price (X)	$\frac{\partial^2 C}{\partial F \partial X} = -\frac{e^{-r_D\tau} N'(d_1)}{X\sigma\sqrt{\tau}}$	$\frac{\partial^2 P}{\partial F \partial X} = -\frac{e^{-r_D\tau} N'(d_1)}{X\sigma\sqrt{\tau}}$
Future Price (F) and Domestic Risk Free Rate (r_D)	$\frac{\partial^2 C}{\partial F \partial r_D} = -\tau \frac{\partial C}{\partial F}$	$\frac{\partial^2 P}{\partial F \partial r_D} = -\tau \frac{\partial P}{\partial F}$
Future Price (F) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial F \partial \sigma} = -e^{-r_D\tau} \frac{N'(d_1) d_2}{\sigma}$	$\frac{\partial^2 P}{\partial F \partial \sigma} = -e^{-r_D\tau} \frac{N'(d_1) d_1}{\sigma}$
Future Price (F) and Time to Expiry (τ)	$\frac{\partial^2 C}{\partial F \partial \tau} = \left(\frac{\sigma}{2\tau}\right) \frac{\partial^2 C}{\partial F \partial \sigma} - r_D \frac{\partial C}{\partial F}$	$\frac{\partial^2 P}{\partial F \partial \tau} = \left(\frac{\sigma}{2\tau}\right) \frac{\partial^2 P}{\partial F \partial \sigma} - r_D \frac{\partial P}{\partial F}$
Strike Price (X) and Domestic Risk Free Rate (r_D)	$\frac{\partial^2 C}{\partial X \partial r_D} = -\tau \frac{\partial C}{\partial X}$	$\frac{\partial^2 P}{\partial X \partial r_D} = -\tau \frac{\partial P}{\partial X}$
Strike Price (X) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial X \partial \sigma} = \frac{\partial C}{\partial \sigma} \frac{d_1}{X\sigma\sqrt{\tau}}$	$\frac{\partial^2 C}{\partial X \partial \sigma} = \frac{\partial C}{\partial \sigma} \frac{d_1}{X\sigma\sqrt{\tau}}$
Strike Price (X) and the Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial X \partial \tau} = \frac{\sigma}{2\tau} \frac{\partial^2 C}{\partial X \partial \sigma} + \frac{r_D}{\tau} \frac{\partial^2 C}{\partial X \partial r_D}$	$\frac{\partial^2 P}{\partial X \partial \tau} = \frac{\sigma}{2\tau} \frac{\partial^2 P}{\partial X \partial \sigma} + \frac{r_D}{\tau} \frac{\partial^2 P}{\partial X \partial r_D}$
Domestic Risk Free Rate (r_D) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial r_D \partial \sigma} = -\tau \frac{\partial C}{\partial \sigma}$	$\frac{\partial^2 P}{\partial r_D \partial \sigma} = -\tau \frac{\partial P}{\partial \sigma}$
Domestic Risk Free Rate (r_D) and The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial r_D \partial \tau} = -\left[\tau \frac{\partial C}{\partial \tau} + C\right]$	$\frac{\partial^2 P}{\partial r_D \partial \tau} = -\left[\tau \frac{\partial P}{\partial \tau} + P\right]$
Annualized Return Volatility (σ) and The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial \sigma \partial \tau} = \frac{1}{2\tau} \frac{\partial C}{\partial \sigma} [d_1 d_2 + 1 - 2 r_D \tau]$	$\frac{\partial^2 P}{\partial \sigma \partial \tau} = \frac{1}{2\tau} \frac{\partial P}{\partial \sigma} [d_1 d_2 + 1 - 2 r_D \tau]$

Merton (1973)
Option Model

Definition of Variables Used in the Merton (1973) Option Formula	
C =	Theoretical Call Option Price
P =	Theoretical Put Option Price
S =	Underlying Asset Spot Price
X =	Exercise (Strike) Price
r_D =	Risk-Free Domestic Interest Rate, Annualized
σ =	Volatility of the Underlying Asset's Returns, Annualized
q =	Dividend Yield of the Underlying Asset
τ =	Time to Expiration, as a fraction of a year
N(x) =	Cumulative Normal Probability Distribution evaluated at x
N´(x) =	Normal Probability Density Function evaluated at x

Intermediate Formulas (d_1 and d_2) for the Merton (1973) Option Pricing Model	
$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r_D - q + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$	$d_2 = \frac{\ln\left(\frac{S}{X}\right) + \left(r_D - q - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$
$N'(d_1) = N'(d_2)\left(\frac{X}{S}\right)e^{-(r_D - q)\tau}$	$N'(d_2) = N'(d_1)\left(\frac{S}{X}\right)e^{(r_D - q)\tau}$
$N''(d_i) = -d_i N'(d_i) \quad i = 1, 2$	

Option Pricing Formula for the Merton (1973) Option Pricing Model Continuously priced cash settled security, continuous dividend yield
$C = e^{-q\tau} S N(d_1) - e^{-r_D\tau} X N(d_2)$
$P = e^{-q\tau} S\{N(d_1) - 1\} - e^{-r_D\tau} X\{N(d_2) - 1\}$

Merton (1973) Option Pricing Model Journal Citation
Merton, Robert C. (Spring 1973). "Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science* **4** (1): 141-183.

Useful Intermediate Derivatives for the Merton (1973) Option Pricing Model	
$\frac{\partial d_1}{\partial S} = \frac{1}{S\sigma\sqrt{\tau}}$	$\frac{\partial d_2}{\partial S} = \frac{1}{S\sigma\sqrt{\tau}}$
$\frac{\partial d_1}{\partial X} = -\frac{1}{X\sigma\sqrt{\tau}}$	$\frac{\partial d_2}{\partial X} = -\frac{1}{X\sigma\sqrt{\tau}}$
$\frac{\partial d_1}{\partial r_D} = \frac{\sqrt{\tau}}{\sigma}$	$\frac{\partial d_2}{\partial r_D} = \frac{\sqrt{\tau}}{\sigma}$
$\frac{\partial d_1}{\partial \sigma} = -\frac{d_2}{\sigma}$	$\frac{\partial d_2}{\partial \sigma} = -\frac{d_1}{\sigma}$
$\frac{\partial d_1}{\partial q} = -\frac{\sqrt{\tau}}{\sigma}$	$\frac{\partial d_2}{\partial q} = -\frac{\sqrt{\tau}}{\sigma}$
$\frac{\partial d_1}{\partial \tau} = \frac{(r_D - q)}{\sigma\sqrt{\tau}} - \frac{d_2}{2\tau}$	$\frac{\partial d_2}{\partial \tau} = \frac{(r_D - q)}{\sigma\sqrt{\tau}} - \frac{d_1}{2\tau}$

First Derivatives of the Merton (1973) Option Pricing Model		
Derivative with Respect to:	**Call Options**	**Put Options**
The Underlying Asset Price (S)	$\frac{\partial C}{\partial S} = e^{-q\tau} N(d_1)$	$\frac{\partial P}{\partial S} = e^{-q\tau} \{N(d_1) - 1\}$
The Option's Strike Price (X)	$\frac{\partial C}{\partial X} = -e^{-r_D\tau} N(d_2)$	$\frac{\partial P}{\partial X} = -e^{-r_D\tau} \{N(d_2) - 1\}$
The Annualized Domestic Risk Free Rate Of Return (r_D)	$\frac{\partial C}{\partial r_D} = -\tau X \frac{\partial C}{\partial X}$	$\frac{\partial P}{\partial r_D} = -\tau X \frac{\partial P}{\partial X}$
The Annualized Instantaneous Return Volatility of the Asset (σ)	$\frac{\partial C}{\partial \sigma} = S\sqrt{\tau} e^{-q\tau} N'(d_1)$	$\frac{\partial P}{\partial \sigma} = S\sqrt{\tau} e^{-q\tau} N'(d_1)$
The Annualized Dividend Yield Of the Underlying Asset (q)	$\frac{\partial C}{\partial q} = -S\tau \frac{\partial C}{\partial S}$	$\frac{\partial P}{\partial q} = -S\tau \frac{\partial P}{\partial S}$
The Option's Time to Expiry (τ)	$\frac{\partial C}{\partial \tau} = \frac{\sigma}{2\tau} \frac{\partial C}{\partial \sigma} + \frac{q}{\tau} \frac{\partial C}{\partial q} + \frac{r_D}{\tau} \frac{\partial C}{\partial r_D}$	$\frac{\partial P}{\partial \tau} = \frac{\sigma}{2\tau} \frac{\partial P}{\partial \sigma} + \frac{q}{\tau} \frac{\partial P}{\partial q} + \frac{r_D}{\tau} \frac{\partial P}{\partial r_D}$

Second Own Derivatives of the Merton (1973) Option Pricing Model		
Derivative with Respect to:	**Call Options**	**Put Options**
The Underlying Asset Price (S)	$\frac{\partial^2 C}{\partial S^2} = \frac{e^{-q\tau} N'(d_1)}{S\sigma\sqrt{\tau}}$	$\frac{\partial^2 P}{\partial S^2} = \frac{e^{-q\tau} N'(d_1)}{S\sigma\sqrt{\tau}}$
The Option's Strike Price (X)	$\frac{\partial^2 C}{\partial X^2} = \frac{e^{-r_D\tau} N'(d_2)}{X\sigma\sqrt{\tau}}$	$\frac{\partial^2 P}{\partial X^2} = \frac{e^{-r_D\tau} N'(d_2)}{X\sigma\sqrt{\tau}}$
The Annualized Domestic Risk Free Rate Of Return (r_D)	$\frac{\partial^2 C}{\partial r_D^2} = -\tau X \frac{\partial^2 C}{\partial X \partial r_D}$	$\frac{\partial^2 P}{\partial r_D^2} = -\tau X \frac{\partial^2 P}{\partial X \partial r_D}$
The Annualized Instantaneous Return Volatility of the Asset (σ)	$\frac{\partial^2 C}{\partial \sigma^2} = \frac{\partial C}{\partial \sigma} \frac{d_1 d_2}{\sigma}$	$\frac{\partial^2 P}{\partial \sigma^2} = \frac{\partial P}{\partial \sigma} \frac{d_1 d_2}{\sigma}$
The Annualized Dividend Yield Of the Underlying Asset (q)	$\frac{\partial^2 C}{\partial q^2} = -S\tau \frac{\partial^2 C}{\partial S \partial q}$	$\frac{\partial^2 P}{\partial q^2} = -S\tau \frac{\partial^2 P}{\partial S \partial q}$
The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial \tau^2} = \frac{\sigma}{2\tau^2} \left[\tau \frac{\partial^2 C}{\partial \sigma \partial \tau} - \frac{\partial C}{\partial \sigma} \right] + Sq \frac{\partial^2 C}{\partial S \partial \tau} - r_D X \frac{\partial^2 C}{\partial X \partial \tau}$	$\frac{\partial^2 P}{\partial \tau^2} = \frac{\sigma}{2\tau^2} \left[\tau \frac{\partial^2 P}{\partial \sigma \partial \tau} - \frac{\partial P}{\partial \sigma} \right] + Sq \frac{\partial^2 P}{\partial S \partial \tau} - r_D X \frac{\partial^2 P}{\partial X \partial \tau}$

Second Cross Derivatives of the Merton (1973) Option Pricing Model

Derivative with Respect to:	Call Options	Put Options
Asset Price (S) and Strike Price (X)	$\frac{\partial^2 C}{\partial S \partial X} = -\frac{e^{-q\tau} N'(d_1)}{X\sigma\sqrt{\tau}}$	$\frac{\partial^2 P}{\partial S \partial X} = -\frac{e^{-q\tau} N'(d_1)}{X\sigma\sqrt{\tau}}$
Asset Price (S) and Domestic Risk Free Rate (r_D)	$\frac{\partial^2 C}{\partial S \partial r_D} = -\tau X \frac{\partial^2 C}{\partial S \partial X}$	$\frac{\partial^2 P}{\partial S \partial r_D} = -\tau X \frac{\partial^2 P}{\partial S \partial X}$
Asset Price (S) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial S \partial \sigma} = -e^{-q\tau} \frac{N'(d_1) d_2}{\sigma}$	$\frac{\partial^2 P}{\partial S \partial \sigma} = -e^{-q\tau} \frac{N'(d_1) d_2}{\sigma}$
Asset Price (S) and Annualized Dividend Yield (q)	$\frac{\partial^2 C}{\partial S \partial q} = -\tau \left[S \frac{\partial^2 C}{\partial S^2} + \frac{\partial C}{\partial S} \right]$	$\frac{\partial^2 P}{\partial S \partial q} = -\tau \left[S \frac{\partial^2 P}{\partial S^2} + \frac{\partial P}{\partial S} \right]$
Asset Price (S) and Time to Expiry (τ)	$\frac{\partial^2 C}{\partial S \partial \tau} = \frac{\sigma}{2\tau} \frac{\partial^2 C}{\partial S \partial \sigma} + \frac{q}{\tau} \frac{\partial^2 C}{\partial S \partial q} + \frac{r_D}{\tau} \frac{\partial^2 C}{\partial S \partial r_D}$	$\frac{\partial^2 P}{\partial S \partial \tau} = \frac{\sigma}{2\tau} \frac{\partial^2 p}{\partial S \partial \sigma} + \frac{q}{\tau} \frac{\partial^2 P}{\partial S \partial q} + \frac{r_D}{\tau} \frac{\partial^2 P}{\partial S \partial r_D}$
Strike Price (X) and Domestic Risk Free Rate (r_D)	$\frac{\partial^2 C}{\partial X \partial r_D} = -\tau \left[X \frac{\partial^2 C}{\partial X^2} + \frac{\partial C}{\partial X} \right]$	$\frac{\partial^2 P}{\partial X \partial r_D} = -\tau \left[X \frac{\partial^2 P}{\partial X^2} + \frac{\partial P}{\partial X} \right]$
Strike Price (X) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial X \partial \sigma} = \frac{e^{-r_D\tau} N'(d_2) d_1}{\sigma}$	$\frac{\partial^2 P}{\partial X \partial \sigma} = \frac{e^{-r_D\tau} N'(d_2) d_1}{\sigma}$
Strike Price (X) and Annualized Dividend Yield (q)	$\frac{\partial^2 C}{\partial X \partial q} = -\frac{e^{-r_D\tau} N'(d_2)\sqrt{\tau}}{\sigma}$	$\frac{\partial^2 P}{\partial X \partial q} = -\frac{e^{-r_D\tau} N'(d_2)\sqrt{\tau}}{\sigma}$
Strike Price (X) and the Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial X \partial \tau} = \frac{\sigma}{2\tau} \frac{\partial^2 C}{\partial X \partial \sigma} + \frac{q}{\tau} \frac{\partial^2 C}{\partial X \partial q} + \frac{r_D}{\tau} \frac{\partial^2 C}{\partial X \partial r_D}$	$\frac{\partial^2 P}{\partial X \partial \tau} = \frac{\sigma}{2\tau} \frac{\partial^2 p}{\partial X \partial \sigma} + \frac{q}{\tau} \frac{\partial^2 P}{\partial X \partial q} + \frac{r_D}{\tau} \frac{\partial^2 P}{\partial X \partial r_D}$
Domestic Risk Free Rate (r_D) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial r_D \partial \sigma} = -\tau X \frac{\partial^2 C}{\partial X \partial \sigma}$	$\frac{\partial^2 P}{\partial r_D \partial \sigma} = -\tau X \frac{\partial^2 P}{\partial X \partial \sigma}$
Domestic Risk Free Rate (r_D) and Annualized Dividend Yield (q)	$\frac{\partial^2 C}{\partial r_D \partial q} = -\tau X \frac{\partial^2 C}{\partial X \partial q}$	$\frac{\partial^2 P}{\partial r_D \partial q} = -\tau X \frac{\partial^2 P}{\partial X \partial q}$
Domestic Risk Free Rate (r_D) and The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial r_D \partial \tau} = -X \left(\tau \frac{\partial^2 C}{\partial X \partial \tau} + \frac{\partial C}{\partial X} \right)$	$\frac{\partial^2 P}{\partial r_D \partial \tau} = -X \left(\tau \frac{\partial^2 P}{\partial X \partial \tau} + \frac{\partial P}{\partial X} \right)$
Annualized Return Volatility (σ) and Annualized Dividend Yield (q)	$\frac{\partial^2 C}{\partial \sigma \partial q} = -S\tau \frac{\partial^2 C}{\partial S \partial \sigma}$	$\frac{\partial^2 P}{\partial \sigma \partial q} = -S\tau \frac{\partial^2 P}{\partial S \partial \sigma}$
Annualized Return Volatility (σ) and Option Time to Expiry (τ)	$\frac{\partial^2 C}{\partial \sigma \partial \tau} = \frac{1}{2\tau} \left[\sigma \frac{\partial^2 C}{\partial \sigma^2} + \frac{\partial C}{\partial \sigma} \right] + \frac{r_D}{\tau} \frac{\partial^2 C}{\partial r_D \partial \sigma} + \frac{q}{\tau} \frac{\partial^2 C}{\partial \sigma \partial q}$	$\frac{\partial^2 P}{\partial \sigma \partial \tau} = \frac{1}{2\tau} \left[\sigma \frac{\partial^2 P}{\partial \sigma^2} + \frac{\partial P}{\partial \sigma} \right] + \frac{r_D}{\tau} \frac{\partial^2 P}{\partial r_D \partial \sigma} + \frac{q}{\tau} \frac{\partial^2 P}{\partial \sigma \partial q}$
Annualized Dividend Yield (q) and Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial q \partial \tau} = \frac{1}{\tau} \left[q \frac{\partial^2 C}{\partial q^2} + \frac{\partial C}{\partial q} \right] + \frac{r_D}{\tau} \frac{\partial^2 C}{\partial r_D \partial q}$ $\frac{\sigma}{2\tau} \frac{\partial^2 C}{\partial \sigma \partial q}$	$\frac{\partial^2 P}{\partial q \partial \tau} = \frac{1}{\tau} \left[q \frac{\partial^2 P}{\partial q^2} + \frac{\partial P}{\partial q} \right] + \frac{r_D}{\tau} \frac{\partial^2 P}{\partial r_D \partial q}$ $\frac{\sigma}{2\tau} \frac{\partial^2 P}{\partial \sigma \partial q}$

Garman-Kohlhagen (1983) Option Model

Definition of Variables Used in the Garman-Kohlhagen (1983) Option Formula	
C =	Theoretical Call Option Price
P =	Theoretical Put Option Price
S =	Underlying Spot Forex Rate
X =	Exercise (Strike) Price
σ =	Volatility of the Underlying Asset's Returns, Annualized
τ =	Time to Expiration, as a fraction of a year
r_{D} =	Risk-Free Domestic Interest Rate, Annualized
r_{F} =	Risk-Free Foreign Interest Rate, Annualized
N(x) =	Cumulative Normal Probability Distribution evaluated at x
N´(x) =	Normal Probability Density Function evaluated at x

Intermediate Formulas (d_1 and d_2) for the Garman-Kohlhagen (1983) Option Pricing Model	
$d_1 = \dfrac{\ln\left(\frac{S}{X}\right)+\left(r_D - r_F + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$	$d_2 = \dfrac{\ln\left(\frac{S}{X}\right)+\left(r_D - r_F - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$
$N'(d_1) = N'(d_2)\left(\dfrac{X}{S}\right)e^{-(r_D - r_F)\tau}$	$N'(d_2) = N'(d_1)\left(\dfrac{S}{X}\right)e^{-(r_D - r_F)\tau}$
$N''(d_i) = -d_i N'(d_i) \quad i = 1, 2$	

Option Pricing Formula for the Garman-Kohlhagen (1983) Option Pricing Model) **Continuously Priced Spot-Traded Foreign Exchange**
$C = e^{-r_F\tau} S N(d_1) - e^{-r_D\tau} X N(d_2)$
$P = e^{-r_F\tau} S\{N(d_1) - 1\} - e^{-r_D\tau} X\{N(d_2) - 1\}$

Garman-Kohlhagen (1983) Option Pricing Model Journal Citation
Garman, Mark B. and Kohlhagen, Steven W. (December 1983). "Foreign Currency Option Values", *Bell Journal of International Money and Finance* **2**: 231-237.

Useful Intermediate Derivatives for the Garman-Kohlhagen (1983) Option Pricing Model	
$\frac{\partial d_1}{\partial S} = \frac{1}{S\sigma\sqrt{\tau}}$	$\frac{\partial d_2}{\partial S} = \frac{1}{S\sigma\sqrt{\tau}}$
$\frac{\partial d_1}{\partial X} = -\frac{1}{X\sigma\sqrt{\tau}}$	$\frac{\partial d_2}{\partial X} = -\frac{1}{X\sigma\sqrt{\tau}}$
$\frac{\partial d_1}{\partial r_D} = \frac{\sqrt{\tau}}{\sigma}$	$\frac{\partial d_2}{\partial r_D} = \frac{\sqrt{\tau}}{\sigma}$
$\frac{\partial d_1}{\partial r_F} = -\frac{\sqrt{\tau}}{\sigma}$	$\frac{\partial d_2}{\partial r_F} = -\frac{\sqrt{\tau}}{\sigma}$
$\frac{\partial d_1}{\partial \sigma} = -\frac{d_2}{\sigma}$	$\frac{\partial d_2}{\partial \sigma} = -\frac{d_1}{\sigma}$
$\frac{\partial d_1}{\partial \tau} = \frac{(r_D - r_F)}{\sigma\sqrt{\tau}} - \frac{d_2}{2\tau}$	$\frac{\partial d_2}{\partial \tau} = \frac{(r_D - r_F)}{\sigma\sqrt{\tau}} - \frac{d_1}{2\tau}$

First Derivatives of the Garman-Kohlhagen (1983) Option Pricing Model		
Derivative with Respect to:	**Call Options**	**Put Options**
The Underlying Spot Foreign Exchange Rate (S)	$\frac{\partial C}{\partial S}=e^{-r_F\tau}N(d_1)$	$\frac{\partial P}{\partial S}=e^{-r_F\tau}\{N(d_1)-1\}$
The Option's Strike Price (X)	$\frac{\partial C}{\partial X}=-e^{-r_D\tau}N(d_2)$	$\frac{\partial P}{\partial X}=-e^{-r_D\tau}\{N(d_2)-1\}$
The Annualized Domestic Risk Free Rate Of Return (r_D)	$\frac{\partial C}{\partial r_D}=-\tau X\frac{\partial C}{\partial X}$	$\frac{\partial P}{\partial r_D}=-\tau X\frac{\partial P}{\partial X}$
The Annualized Instantaneous Return Volatility of the Asset (σ)	$\frac{\partial C}{\partial \sigma}=Se^{-r_F\tau}\sqrt{\tau}N(d_1)$	$\frac{\partial P}{\partial \sigma}=Se^{-r_F\tau}\sqrt{\tau}N(d_1)$
The Annualized Foreign Risk Free Rate of Return (r_F)	$\frac{\partial C}{\partial r_F}=-S\tau\frac{\partial C}{\partial S}$	$\frac{\partial P}{\partial r_F}=-S\tau\frac{\partial P}{\partial S}$
The Option's Time to Expiry (τ)	$\frac{\partial C}{\partial \tau}=\frac{\sigma}{2\tau}\frac{\partial C}{\partial \sigma}+Sr_F\frac{\partial C}{\partial S}+\frac{r_D}{\tau}\frac{\partial C}{\partial r_D}$	$\frac{\partial P}{\partial \tau}=\frac{\sigma}{2\tau}\frac{\partial P}{\partial \sigma}+Sr_F\frac{\partial P}{\partial S}+\frac{r_D}{\tau}\frac{\partial P}{\partial r_D}$

Second Own Derivatives of the Garman-Kohlhagen (1983) Option Pricing Model		
Derivative with Respect to:	**Call Options**	**Put Options**
The Underlying Spot Foreign Exchange Rate (S)	$\frac{\partial^2 C}{\partial S^2}=\frac{e^{-r_F\tau}N'(d_1)}{S\sigma\sqrt{\tau}}$	$\frac{\partial^2 P}{\partial S^2}=\frac{e^{-r_F\tau}N'(d_1)}{S\sigma\sqrt{\tau}}$
The Option's Strike Price (X)	$\frac{\partial^2 C}{\partial X^2}=\frac{e^{-r_D\tau}N(d_2)}{X\sigma\sqrt{\tau}}$	$\frac{\partial^2 P}{\partial X^2}=\frac{e^{-r_D\tau}N(d_2)}{X\sigma\sqrt{\tau}}$
The Annualized Domestic Risk Free Rate Of Return (r_D)	$\frac{\partial^2 C}{\partial {r_D}^2}=-\tau X\frac{\partial^2 C}{\partial X\partial r_D}$	$\frac{\partial^2 P}{\partial {r_D}^2}=-\tau X\frac{\partial^2 P}{\partial X\partial r_D}$
The Annualized Instantaneous Return Volatility of the Asset (σ)	$\frac{\partial^2 C}{\partial \sigma^2}=\frac{\partial C}{\partial \sigma}\frac{d_1 d_2}{\sigma}$	$\frac{\partial^2 P}{\partial \sigma^2}=\frac{\partial P}{\partial \sigma}\frac{d_1 d_2}{\sigma}$
The Annualized Foreign Risk Free Rate of Return (r_F)	$\frac{\partial^2 C}{\partial {r_F}^2}=-S\tau\frac{\partial^2 C}{\partial S\partial r_F}$	$\frac{\partial^2 P}{\partial {r_F}^2}=-S\tau\frac{\partial^2 P}{\partial S\partial r_F}$
The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial \tau^2}=\frac{\sigma}{2\tau^2}\left[\tau\frac{\partial^2 C}{\partial\sigma\partial\tau}-\frac{\partial C}{\partial\sigma}\right]+Sr_F\frac{\partial^2 C}{\partial S\partial\tau}-r_D X\frac{\partial^2 C}{\partial X\partial\tau}$	$\frac{\partial^2 P}{\partial \tau^2}=\frac{\sigma}{2\tau^2}\left[\tau\frac{\partial^2 P}{\partial\sigma\partial\tau}-\frac{\partial P}{\partial\sigma}\right]+Sr_F\frac{\partial^2 P}{\partial S\partial\tau}-r_D X\frac{\partial^2 P}{\partial X\partial\tau}$

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Second Cross Derivatives of the Garman-Kohlhagen (1983) Option Pricing Model		
Derivative with Respect to:	**Call Options**	**Put Options**
Spot FX Rate (S) and Strike Price (X)	$\frac{\partial^2 C}{\partial S \partial X} = -\frac{e^{-r_F \tau} N'(d_1)}{X \sigma \sqrt{\tau}}$	$\frac{\partial^2 P}{\partial S \partial X} = -\frac{e^{-r_F \tau} N'(d_1)}{X \sigma \sqrt{\tau}}$
Spot FX Rate (S) and Domestic Risk Free Rate (r_D)	$\frac{\partial^2 C}{\partial S \partial r_D} = -\tau X \frac{\partial^2 C}{\partial S \partial X}$	$\frac{\partial^2 P}{\partial S \partial r_D} = -\tau X \frac{\partial^2 P}{\partial S \partial X}$
Spot FX Rate (S) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial S \partial \sigma} = -e^{-r_F \tau} \frac{N'(d_1) d_2}{\sigma}$	$\frac{\partial^2 C}{\partial S \partial \sigma} = -e^{-r_F \tau} \frac{N'(d_1) d_2}{\sigma}$
Spot FX Rate (S) and Foreign Risk Free Rate (r_F)	$\frac{\partial^2 C}{\partial S \partial r_F} = -\tau \left[S \frac{\partial^2 C}{\partial S^2} + \frac{\partial C}{\partial S} \right]$	$\frac{\partial^2 P}{\partial S \partial r_F} = -\tau \left[S \frac{\partial^2 P}{\partial S^2} + \frac{\partial P}{\partial S} \right]$
Spot FX Rate (S) and Time to Expiry (τ)	$\frac{\partial^2 C}{\partial S \partial \tau} = \frac{\sigma}{2\tau} \frac{\partial^2 C}{\partial S \partial \sigma} + \frac{r_F}{\tau} \frac{\partial^2 C}{\partial S \partial r_F} + \frac{r_D}{\tau} \frac{\partial^2 C}{\partial S \partial r_D}$	$\frac{\partial^2 P}{\partial S \partial \tau} = \frac{\sigma}{2\tau} \frac{\partial^2 p}{\partial S \partial \sigma} + \frac{r_F}{\tau} \frac{\partial^2 P}{\partial S \partial r_F} + \frac{r_D}{\tau} \frac{\partial^2 P}{\partial S \partial r_D}$
Strike Price (X) and Domestic Risk Free Rate (r_D)	$\frac{\partial^2 C}{\partial X \partial r_D} = -\tau \left[X \frac{\partial^2 C}{\partial X^2} + \frac{\partial C}{\partial X} \right]$	$\frac{\partial^2 P}{\partial X \partial r_D} = -\tau \left[X \frac{\partial^2 P}{\partial X^2} + \frac{\partial P}{\partial X} \right]$
Strike Price (X) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial X \partial \sigma} = \frac{e^{-r_D \tau} N'(d_2) d_1}{\sigma}$	$\frac{\partial^2 P}{\partial X \partial \sigma} = \frac{e^{-r_D \tau} N'(d_2) d_1}{\sigma}$
Strike Price (X) and Foreign Risk Free Rate (r_F)	$\frac{\partial^2 C}{\partial X \partial r_F} = -\frac{e^{-r_D \tau} N'(d_2) \sqrt{\tau}}{\sigma}$	$\frac{\partial^2 P}{\partial X \partial r_F} = -\frac{e^{-r_D \tau} N'(d_2) \sqrt{\tau}}{\sigma}$
Strike Price (X) and the Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial X \partial \tau} = \frac{\sigma}{2\tau} \frac{\partial^2 C}{\partial X \partial \sigma} + \frac{r_F}{\tau} \frac{\partial^2 C}{\partial X \partial r_F} + \frac{r_D}{\tau} \frac{\partial^2 C}{\partial X \partial r_D}$	$\frac{\partial^2 P}{\partial X \partial \tau} = \frac{\sigma}{2\tau} \frac{\partial^2 P}{\partial X \partial \sigma} + \frac{r_F}{\tau} \frac{\partial^2 P}{\partial X \partial r_F} + \frac{r_D}{\tau} \frac{\partial^2 P}{\partial X \partial r_D}$
Domestic Risk Free Rate (r_D) and Annualized Return Volatility (σ)	$\frac{\partial^2 C}{\partial r_D \partial \sigma} = -\tau X \frac{\partial^2 C}{\partial X \partial \sigma}$	$\frac{\partial^2 P}{\partial r_D \partial \sigma} = -\tau X \frac{\partial^2 P}{\partial X \partial \sigma}$
Domestic Risk Free Rate (r_D) and Foreign Risk Free Rate (r_F)	$\frac{\partial^2 C}{\partial r_D \partial r_F} = -\tau X \frac{\partial^2 C}{\partial X \partial r_F}$	$\frac{\partial^2 P}{\partial r_D \partial r_F} = -\tau X \frac{\partial^2 P}{\partial X \partial r_F}$
Domestic Risk Free Rate (r_D) and The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial r_D \partial \tau} = -X \left(\tau \frac{\partial^2 C}{\partial X \partial \tau} + \frac{\partial C}{\partial X} \right)$	$\frac{\partial^2 P}{\partial r_D \partial \tau} = -X \left(\tau \frac{\partial^2 P}{\partial X \partial \tau} + \frac{\partial P}{\partial X} \right)$
Annualized Return Volatility (σ) and Foreign Risk Free Rate (r_F)	$\frac{\partial^2 C}{\partial \sigma \partial r_F} = -S \tau \frac{\partial^2 C}{\partial S \partial \sigma}$	$\frac{\partial^2 P}{\partial \sigma \partial r_F} = -S \tau \frac{\partial^2 P}{\partial S \partial \sigma}$
Annualized Return Volatility (σ) and Option Time to Expiry (τ)	$\frac{\partial^2 C}{\partial \sigma \partial \tau} = \frac{1}{2\tau} \left[\sigma \frac{\partial^2 C}{\partial \sigma^2} + \frac{\partial C}{\partial \sigma} \right] + \frac{r_D}{\tau} \frac{\partial^2 C}{\partial r_D \partial \sigma} + \frac{r_F}{\tau} \frac{\partial^2 C}{\partial \sigma \partial r_F}$	$\frac{\partial^2 P}{\partial \sigma \partial \tau} = \frac{1}{2\tau} \left[\sigma \frac{\partial^2 P}{\partial \sigma^2} + \frac{\partial P}{\partial \sigma} \right] + \frac{r_D}{\tau} \frac{\partial^2 P}{\partial r_D \partial \sigma} + \frac{r_F}{\tau} \frac{\partial^2 P}{\partial \sigma \partial r_F}$
Foreign Risk Free Rate (r_F) and The Option's Time to Expiry (τ)	$\frac{\partial^2 C}{\partial r_F \partial \tau} = \frac{1}{\tau} \left[r_F \frac{\partial^2 C}{\partial r_F^2} + \frac{\partial C}{\partial r_F} \right] + \frac{\sigma}{2\tau} \frac{\partial^2 C}{\partial \sigma \partial r_F} + \frac{r_D}{\tau} \frac{\partial^2 C}{\partial r_D \partial r_F}$	$\frac{\partial^2 P}{\partial r_F \partial \tau} = \frac{1}{\tau} \left[r_F \frac{\partial^2 P}{\partial r_F^2} + \frac{\partial P}{\partial r_F} \right] + \frac{\sigma}{2\tau} \frac{\partial^2 P}{\partial \sigma \partial r_F} + \frac{r_D}{\tau} \frac{\partial^2 P}{\partial r_D \partial r_F}$